

# **Relation Philosophy *of* Mathematics, Science, and Mind**

by

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## ***Table of Contents***

Introduction.....	<a href="#">vi</a>
<b><u>PART ONE</u></b>	
<a href="#">Relations as a Foundation for Mathematics</a>	
Preface .....	<a href="#">1</a>
1. Primitive Concepts.....	<a href="#">4</a>
2. Sets.....	<a href="#">15</a>
3. Three Kinds of Mathematical Meaning .....	<a href="#">36</a>
4. Mathematical Reasoning.....	<a href="#">59</a>
5. Foundations.....	<a href="#">74</a>
6. Some Theorems .....	<a href="#">91</a>
<b><u>PART TWO</u></b>	
<a href="#">Intensional Philosophy of Mathematics</a>	
Preface .....	<a href="#">96</a>
7. Hekergy.....	<a href="#">98</a>
8. The Ontological Argument .....	<a href="#">105</a>
<b><u>PART THREE</u></b>	
<a href="#">Intensional Philosophy of Science</a>	
9. Problems in Philosophy of Science.....	<a href="#">120</a>
10. Observation.....	<a href="#">126</a>
11. The Leibniz-Russell Theory.....	<a href="#">144</a>
12. Application of the Leibniz-Russell Theory .....	<a href="#">148</a>
<b><u>PART FOUR</u></b>	
<a href="#">Intensional Philosophy of Mind</a>	
13. Mind.....	<a href="#">160</a>
14. The Oge .....	<a href="#">176</a>
15. Gods.....	<a href="#">196</a>
16. Rational Mind .....	<a href="#">200</a>

## REFERENCE

Synopsis.....	<u>206</u>
Glossary of Symbols.....	<u>239</u>
Glossary .....	<u>241</u>
Index .....	<u>290</u>

*Do not multiply entities beyond necessity, but,  
also,  
do not reduce them beyond necessity.*

## ***Introduction***

This book is a philosophical work written primarily for mathematicians and scientists, as well as lovers of mathematics and lovers of science. It comes to praise these subjects, not to bury them.

Part One contains a challenge to the contemporary practice of making set theory the foundation of mathematics. It is argued that because the set theoretic definition of relations is circular — it presupposes many relations, such as set-membership and subset — and because mathematics is primarily our language of relations, it is better to make relations primitive and define sets and set theory by means of them. Doing this has a number of interesting results, one of which is the clear definition of set intensions by means of relations. This leads to the distinction of three kinds of set theory: intensional set theory, in which sets have both intensions and extensions; extensional set theory, in which some sets have no intensions; and nominal set theory, in which some sets have neither intensions nor extensions. And in turn this leads to the distinction between intensional, extensional, and nominal meaning in mathematics, and the discovery that only nominal meaning can produce paradox and inconsistency, while only intensional meaning can produce axiom generosity.

Part Two looks at the major problems in philosophy of mathematics. In the process of trying to solve them, there is defined a property of relations called hekergy, which is a generalisation, to relations, of the concept of negative entropy. It is later shown that the hekergy of a relation might be called the absolute value of that relation, the apprehension of which by a person is a subjective human value, such as truth, beauty, or goodness. The power and beauty of mathematics then can be explained by means of hekergy. Part Two then examines the extent

of intensional mathematics and shows that only one intensional mathematical system exists, and that this one does so because it is the best of all possible mathematical systems. Furthermore, the world described by true theoretical science partly describes this best, so that intensional mathematics is essentially applied mathematics.

This in turn leads to philosophy of science, in Part Three. The major problems in this field are examined and it is shown that they may be solved provided only that the Leibniz-Russell theory of perception is accepted. This theory is difficult because it denies an almost immutable common sense belief, a belief so basic that most people never state it, let alone question it, but the theory deserves serious consideration because of its power in philosophy of science: it deals with observation and the nature of the empirical — the foundations of science. This theory also solves many philosophical problems of perception, critical in any treatment of empirical observation.

In Part Four a theory of mind is developed, with the aim of explaining how a mind may think mathematically and how knowledge of the best of all possible mathematical systems may be obtained. Such a theory is possible only because of points made earlier, such as the reality of relations, the nature of hekerger, and the Leibniz-Russell theory of perception.

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# PART ONE

## Relations as a Foundation for Mathematics

### *Preface*

Our first object is to distinguish three kinds of mathematical meaning, called intensional, extensional, and nominal, and to show that all mathematical necessity and axiom generosity occur only with intensional meaning, and all contradiction and paradox occur only with nominal meaning. We here take relations to be primitive, and relations and their properties are intensional meanings. Relations necessarily define sets of their terms — their *relata* — and also sets of their properties, and these sets are the basis of extensional meanings. And nominal meanings are meanings by verbal analogy with intensional and extensional meanings.

Three preliminary points need to be made about relations: the circularity of the usual set theoretic definition of relations, the non-existence of many relations, including monadic relations, and the problem of their perceptibility.

The usual definition of a relation as a subset of a Cartesian product<sup>1</sup> is circular in that it logically presupposes many relations prior to the definition of relation, such as the relations included in the rules that specify the subsets of the Cartesian products, relations such as set-membership and subset, the ordering of

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<sup>1</sup> The Cartesian product of two sets  $A$  and  $B$ , symbolised  $A \times B$ , is the set of all ordered sets  $(a,b)$  such that  $a \in A$  and  $b \in B$ :  $A \times B = \{(a,b): a \in A \ \& \ b \in B\}$ . The definition may be extended to any number of sets.

ordered sets, and polyadic relations such as truth-functions, and argument forms. Because of this we have to say that, logically, set theory is an unsatisfactory basis for relations and functions. The alternative is to make relations definitionally primitive, which is one of the objects of the present book.

Second, there is an old objection to the reality of many relations: namely, their extravagant multiplication. For example, if *improper part* is a real relation then everything is an improper part of itself so that, given one instance of *improper part*, there exists a second relation of *improper part* which has the first as its term; and this second is also an improper part of itself, thereby producing a third *improper part*, and so on without end. *Self-similarity* and *self-identity* are other examples of this kind. Again, if there is a relation *term of* between a relation and each of its terms, then another infinite sequence of superfluous relations results; this is true of any relation that holds between any relation and any of its terms. Another infinite multiplication occurs with relations of *similarity* and *dissimilarity*. If similarities are real relations then any two similarities are similar, and this last similarity is similar to every other similarity; any two dissimilarities are similar in the same way, and a pair of a similarity and a dissimilarity are dissimilar. We need to deny this infinite extravagance, and we achieve this by appeal to Occam's Razor. We deny the existence of any monadic relations, and of any relations such as *term of* which multiply extravagantly and uselessly; they are what will be called purely nominal relations (Def. 1.7): names, of relations, which have no reference.

Third, it is sometimes questioned whether relations can be real, given the seeming difficulty of perceiving them. To take a concrete example, if you have a cup of coffee then clearly the coffee is *in* the cup; so we have three things: the cup, the coffee, and the relation *in*. The cup is white, hard, shiny, and hollow; the coffee is hot, brown, sweet, and liquid; but the *in* does not have any colour, taste, texture, temperature, or any other concrete

property. So if the *in* does not have any looks or feels, how can we perceive it? It is very tempting — and many major philosophers have fallen for this temptation — to say that because of this empirical relations must be unreal: they are merely things of the mind, an unconscious means of ordering phenomena. However, this cannot work. We have to say that the coffee is really in the cup, the relation *in* is real, because if this were not so how could you drink your coffee? In fact, the answer to this problem is that relations are real entities in the world around us, but they do not have any looks, feels, etc. because these are all concrete properties and relations are abstract entities, they do not have any concrete properties. This is a disturbing conclusion for earthy people for whom only the concrete is real, hence nothing abstract is real, but for mathematicians it should be no problem.

# 1. Primitive Concepts

We begin with one main primitive concept, **relation**; secondary primitive concepts will be various particular relations.

Relations have three essential characteristics: they are **simple** entities, they have both **intrinsic** and **extrinsic properties**, and one of the intrinsic properties is an **adicity**. The intrinsic properties of a relation are what determine the **kind** of relation that it is, and the extrinsic properties determine the **instance** of that kind.

As we saw in the Preface, there is no relation *term of* between a relation and any of its terms, because of extravagant multiplication. However, *term of* must be meaningful, since it is a matter of fact whether a given entity is a term of a given relation, or not. As it turns out, *term of* is an extrinsic property. Extrinsic properties are defined by means of skew-separability:

**Def. 1.1**      Whatever A and B may be, A is **skew-separable** from B if A can exist without B, but B cannot exist without A.

For example, if a whole, W, has a part, P, then P is skew-separable from W.

**Def. 1.2**      If a relation R is skew-separable from another relation, S, then R is a **lower extrinsic property** of S, and S is an **upper extrinsic property** of R. And a property of a relation is an **intrinsic property**<sup>2</sup> of that relation if the relation and the property are inseparable

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<sup>2</sup>Unfortunately the word *intrinsic* is ambiguous, in that it also refers to the primitive relation *part of*: We want to say that a part is intrinsic to the whole and that a member is intrinsic to the set (although a term is not in this sense intrinsic

In ordinary language, if  $R$  is a lower extrinsic property of  $S$  then  $R$  is a term of  $S$  and  $S$  has  $R$  as a term. So the totality of lower extrinsic properties of  $S$  is all of the terms of  $S$ , and the totality of upper extrinsic properties of  $R$  is all of the relations of which  $R$  is a term. We will also include among the upper extrinsic properties of  $R$  all the other terms of the relations of which  $R$  is a term, in order to conform with standard usage.

For example, if Schrödinger's cat is alive-dead in the closed box, the cat has the intrinsic property of being alive-dead and the upper extrinsic property of being in the box; the box has the intrinsic property of being closed and the upper extrinsic property of having an alive-dead cat in it; and the relation *in* has the intrinsic property of being asymmetric and the lower extrinsic properties of a cat and a box.

Although these definitions are needed for formal precision, they are verbally awkward and, unless otherwise needed, will be replaced with equivalent ordinary language expressions, as follows: the lower extrinsic properties of a relation are its **terms**; the upper extrinsic properties of a relation are called its **extrinsic properties**; and the intrinsic properties of a relation are called simply its **properties**. Also in conformity with ordinary usage, we will say that a relation *has*, or *possesses*, (see Def. 2.31) terms and properties, even though such having and possessing are not relations.

The simplicity of a relation is a universal characteristic of relations. This simplicity means both that a relation has no parts and that it is a unity; it is not compounded out of other relations; nor is it a logical construct, manufactured out of its terms, or out of

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to the relation, since a relation has no parts), even though these two are lower extrinsic, to their whole or set. So in what follows we will use both meanings of *intrinsic* as required and the meaning intended will be specified if it is not clear from the context.

its properties. The multiplicity of terms and of properties of a relation must not be confused with its simplicity: the terms are many and the properties are usually many, but the relation is one. Indeed, one might say that the unity of a relation is due to its having only one part, the relation itself.

A term of a relation is always either another relation or a property of a relation. (An exception to this is relations which have concrete qualities or concrete objects as their terms: empirical relations or relations in the imagination; however, these do not usually belong in mathematics and will be ignored until Part Four.)

The properties of a relation are the terms of similarity and dissimilarity relations (see Def. 1.20).

A second universal property of relations is that every relation has a particular **adicity**, which is an intrinsic property of that relation. The adicity of a relation is, speaking metalinguistically, the number of terms that it has. We repeat that there are no monadic relations, so the least adicity of a relation is two. We thus speak of **dyadic**, **triadic**, **tetradic**, **polyadic**, etc., relations. We will later (**Chapter 5**) define intensional number in terms of adicity, so adicity is here logically prior to number, and primitive.

At first sight there seems to be a third universal property of relations: as entities, they are all abstract — as discussed in the Preface. But this is really the absence of certain properties: no relations have any concrete properties such as colour or solidity.

A relation cannot be one of its own terms.

When a relation comes into existence it is said to **emerge**, and when it ceases to exist it is said to **submerge**. If certain relations exist then it is possible that they may be terms of other emergent relations, and these other relations may in their turn be terms of yet other emergent relations, and so on for many higher and higher levels of emergence; such emergence is called **cascading emergence**. It is characteristic of good axiom sets that concepts and theorems emerge cascadingly from them.

Since a relation may have terms of its terms, for lower and lower levels, we distinguish between the immediate terms of a relation, called its **ordinate terms**, and all the terms of its terms, terms of terms of terms, and on down to the lowest level, all of which are its **subordinate terms** (Def. 5.15). We will argue later that there is both a lowest level of relation (see Def. 5.19ff.) and a highest level.

Because the terms of relations are generally other relations, a relation will be symbolised here by a capital letter, and its terms by small caps, this being the closest possible to the contemporary convention that relations are symbolised by upper case letters and their terms by lower case letters. For example, aRb indicates that the relation R has the relations a and b as terms. Thus a particular instance of a relation may be symbolised by either an upper case letter or a small cap, depending on whether its status is being shown to be a relation or a term of a relation; if it should occur more than once in an expression, both as a relation and as a term, then it usually will be written as a small cap.

\* \* \*

The following is a preliminary classification of relations.

**Def. 1.3** A primitive relation which exists independently of any mind is a **real relation**.

**Def. 1.4** A primitive relation which exists within a mind is an **ideal relation**, also called an **abstract idea**.

What is meant by *mind* here will be discussed in Part Four.

**Def. 1.5** A relation which is either real or ideal is a **genuine relation**, also called an **intensional relation**. When the

word relation is used here without qualification it refers to a genuine relation.

**Def. 1.6** A subset of a Cartesian product is an **extensional relation**.

We will mostly not be concerned with extensional relations.

**Def. 1.7** A grammatical form of words that indicates a relation is called a **nominal relation**; if it does not refer to either a genuine relation or an extensional relation then it is called a **purely nominal relation**.

The meanings of descriptions of non-existent relations are meanings by linguistic analogy, as will be explained later.

No extensional relations or purely nominal relations are genuine relations.

\* \* \*

Although not strictly necessary, we first define some familiar concepts, for the sake of completeness.

**Def. 1.8** The **inverse**,  $S$ , of a dyadic relation  $R_{ab}$  is  $S_{ba}$ .

$R_{ab}$  and  $S_{ba}$  are two descriptions of one relation, so that the difference between a dyadic relation and its inverse is only nominal. For example, if one number  $a$  is *less than* another number  $b$  then  $b$  is *greater than* the number  $a$ , and this *greater than* is the inverse of this *less than*; but there is only one relation of relative size, or inequality, between  $a$  and  $b$ , which may be described in two ways: stating  $a$  first, or stating  $b$  first.

**Def. 1.9** A dyadic relation  $Rab$  is **asymmetric** if it has a mathematical sense, as a vector has sense; it is called **symmetric** if it does not have a sense.

Because of its sense an asymmetric relation  $Rab$  is dissimilar (Def. 1.20) to its inverse,  $Sba$ ; and it is symmetric if it is not asymmetric; but this explanation, while illuminating, is not definitive, since *inverse* and *symmetric* have nominal meaning only: *symmetric* signifies only the absence of asymmetry. So asymmetry is a property of some dyadic relations, whereas symmetry and the inverse of a relation are not relational properties.

**Def. 1.10** A dyadic relation,  $R$ , is **transitive** if, given  $Rab$  and  $Rbc$ , it is true that  $Rac$ . and it is **intransitive** if it is not transitive.

**Def. 1.11** A dyadic relation  $R$  is **reflexive** if, of any possible term  $a$  of  $R$ , it is true that  $Raa$ .  $R$  is not monadic in such a case because the expression  $Raa$  means that two distinct instances (Def. 2.28) of  $a$  are the terms of  $R$ . If  $a$  should be one specific instance then  $Raa$  would be monadic and so nominal only.

\* \* \*

We next consider four special relations which are needed in later discussion: they are *possibility*, *identity*, *similarity* and *dissimilarity*.

**Def. 1.12** A **possibility relation** is characterised by having one special term called the **antecedent**; all its other terms are called **consequents**. Each consequent is a possible emergent relation, given the antecedent; and, as possibilities, the consequents are mutually exclusive, and exhaustive.

This definition may be enlarged to include several terms in an antecedent in one possibility relation; for example, a binary operation has an antecedent of two terms.

**Def. 1.13**      The number of consequents of a possibility relation is its **degree of possibility**.

A possibility relation is here symbolised by  $\vee$ , and the disjunction of its consequents by a vertical stroke,  $|$ ; thus if  $a$  is the antecedent of consequents  $c_1$  to  $c_n$ , this is symbolised by  $a\vee(c_1|c_2|\dots c_n)$ , which means that given  $a$ , one and only one of the  $c_1$  to  $c_n$  will emerge. If  $a\vee(c_1|c_2|\dots c_n)$  we say that  $a$  **allows**  $c_1$ ,  $a$  **allows**  $c_2$ , etc.

**Def. 1.14**      A **necessity** is any possibility relation having a degree of possibility of one, a singular possibility.

**Def. 1.15**      A **bipossibility** is any possibility relation having a degree of possibility of two.

**Def. 1.16**      A **contingency** is any possibility relation having a degree of possibility greater than one, a plural possibility; its degree of possibility is also called its **degree of contingency**.

**Def. 1.17**      An **impossibility** might be defined as a zero possibility, which would make it a possibility relation of degree zero; but because there are no monadic relations, an impossibility is not a genuine relation, it is a purely nominal relation.

Necessity is the basis of many mathematical functions, and of mathematical reasoning; bipossibility is the basis of complementary relations; and a contingency is a basis of

probability theory. We next look at each of these three degrees of possibility in greater detail. In each case we consider intensional meaning only: the extensional and nominal meanings of the various concepts will be discussed in Chapter 3.

In pure mathematics relations of necessity occur as some functions, mappings, operations, and transforms, since, given any argument of any of these, the value is a singular possibility: a necessity exists between each antecedent, or argument, and its consequent, or value, as a singular possibility; the value *cannot* be otherwise. Thus  $2^{22}$  is *necessarily* 4,194,304. Similarly, a binary operation is a triadic relation which is a function between a pair of arguments, and a value which is a singular possibility given those

arguments. Thus  $\sqrt{8} + \sqrt{18}$  is *necessarily*  $\sqrt{50}$ . Not all functions in mathematics are necessity relations: some are only correlations (see Defs 3.9 and 3.10).

Necessities also occur as logical necessities, in which the truth of a set of premises necessitates the truth of their conclusions: given the truth of the premises, there is only one possibility for the truth-value of each conclusion, namely, truth. Truth is a relation, as we shall see, and the singular possibility between truths is the necessity known as validity.

A necessity relation is here symbolised by  $q$ ; its inverse by  $r$ ; by  $s$  if it is symmetric; and by  $Q$ ,  $R$ , or  $S$  if it does not exist. If  $AqB$  we say that  $A$  **necessitates**  $B$ .

Bipossibilities, our second kind of possibility relation, have only two consequents. They occur with pairs of relations, such as *similar* and *dissimilar*, *true* and *false*, and *inside* and *outside*. Given the requisite antecedent, one of the pair has to emerge, and thereby excludes the emergence of the other. Kinds of relations which occur in such mutually exclusive pairs of consequents are called **intensional complements** of each other, and will be dealt with later (Def. 2.27).

Contingencies and their degrees, our third kind of possibility relation, are a basis of probability theory.

**Def. 1.18** If an antecedent  $a$  of a contingency relation of degree  $c$  has as one of its consequents the term  $c$ , and each contingency is weighted equally, then the **probability** of  $c$ , given  $a$ , is  $1/c$ ; and if there are  $n$  of the possibilities that are equivalent in some respect then the probability of any one of these equivalent possibilities, given  $a$ , is  $n/c$ .

The limits of probabilities are 0 and 1, but these mark an open interval, not a closed one; this is because there is no degree of possibility of zero, other than nominally, because possibility relations are never monadic; and necessity is not a contingency.

Our second special relation, identity, is defined as:

**Def. 1.19** Two or more symbols, words, names, or descriptions which between them have only one reference are said to be **identical**.

Thus identity is a relation between these words and their one reference. For example, a relation and its inverse are identical; and when we say that the cube root of eight and the even prime are identical, we mean that both these descriptions describe one and the same number. Identity is a linguistic relation because it relates language and reference; so far as the reference alone is concerned, there is only the one reference and no relation of identity. Identity will be symbolised by  $=$ .

Our third and fourth special relations are similarity and dissimilarity.

**Def. 1.20** **Similarity** and **dissimilarity** are dyadic, symmetric, relations which are otherwise primitive — although we can say metalinguistically that their names have the usual meanings, for

which synonyms are *same* and *different*, *like* and *unlike*, and *resembling* and *non-resembling*.

Similarities and dissimilarities will be called compoundable relations (Def. 4.1) and their terms are properties of relations rather than relations. It is a deficiency of the present work that there is no clear principle for knowing which similarities and dissimilarities exist. As we saw in the Preface, they multiply extravagantly, in that any two similarities are similar, as are any two dissimilarities, and any pair of a similarity and a dissimilarity are dissimilar. So, invoking Occam's Razor, this extravagant multiplication must be denied. One way of doing this is to say that similarities and dissimilarities are consequents of a bipossibility relation: given any pair of relational properties as the antecedent of the bipossibility, the consequents are either similarity or dissimilarity. This bipossibility is called a **comparison**, and occurs in computers and minds; because of it we can compare any two similarities and get a similarity, but this third similarity does not exist unless the comparison is made — and thus we obliterate the extravagance. However, there are other situations where we want to say that similarities and dissimilarities exist, without comparisons being made: for example, a *boundary* is a series of contiguous dissimilarities. So we might add the criterion of contiguity to the existence of dissimilarities, were it not for yet other cases of their existence that are not contiguous. A *change*, for example is a *dissimilarity* in parallel with a *duration*, and the absence of change, or *stasis*, is a *similarity* in parallel with a *duration*. So we have to say, for want of anything better, that the existence of real similarities and dissimilarities is determined by both Occam's Razor and its converse: we deny the extravagance by not multiplying them beyond necessity — although at the same time we also multiply their existence up to necessity.

Similarity will be symbolised here by *t*, and dissimilarity by *T*: symbols which will be easier to remember if their origin is

explained. We will later (Def. 4.6) define the intensional truth and falsity of ideal relations by means of their similarity and dissimilarity to real relations. Falsity is symbolised by the tilde,  $\sim$ , so truth will here be symbolised by the tilde rotated through a right angle,  $\perp$ , since such rotation of a symmetric symbol is like negation in that double application of the operation is the identity operation: double negation is affirmation. Since truth and falsity will be special cases of similarity and dissimilarity, these latter are symbolised in parallel fashion by  $\sim$  and  $\perp$ .

Finally in this chapter we mention that not all necessities are relations. Some extrinsic properties include necessity. For example, given the concept of mathematical existence, if a relation exists then its terms necessarily exist, because of the skew-separability of terms from relations; so we can say that the existence of the terms is *extrinsic necessary existence*, given the existence of the relation. In the same way emergence of a relation is extrinsic necessary existence. We go into greater detail on this in Chapter 8.

## 2. Sets

Every relation determines three sets; the set of the terms of the relation, the set of its intrinsic properties, and the set of its extrinsic properties. These may be thought of as natural sets, since they are essential to any discussion of relations, but we will mostly call them intensional sets:

**Def. 2.1**      An **intensional set** is a plurality united by a relation.

Although there is no grammatical difference between a set and a plurality — we speak of *a* plurality and *the* plurality, just as we do of sets — they are fundamentally different: a plurality is many and a set is one. This difference must be kept in mind, since ordinary language will obscure it.

We follow the usual convention of using upper case italic letters to represent sets: *A*, *B*, *C*, etc.

**Def. 2.2**      A **member** of an intensional set is any one element of the unified plurality; the relation between a member and its set is the relation of **set-membership**.

**Def. 2.3**      The **extension** of an intensional set is its plurality.

**Def. 2.4**      The **intension** of an intensional set is the commonality (Def. 2.17 ff.) of its plurality: those properties, intrinsic or extrinsic, possessed by all and only the members of the set; as such the intension is an extrinsic property of each member.

**Def. 2.5**      The **function every** is the necessity relation which has intensions as its arguments and intensional sets as its values; its inverse is the **function any**.

For the definition of intensional function, see Def. 3.7.  
Because the function *every* has an inverse, every intension has only one intensional set and every intensional set has only one kind of intension.

- Def. 2.6**      The **term set** of a relation  $R$  is the intensional set consisting of every term of  $R$ ; it is symbolised by the same letter, italic:  $R$ .
- Def. 2.7**      The **intrinsic property set**, or simply **property set**, of a relation  $R$  is the intensional set of every intrinsic property of  $R$ ; it is symbolised by the same letter, as a capped small cap:  $R$ .
- Def. 2.8**      The **extrinsic property set** of a relation  $R$  is the intensional set of every upper extrinsic property of  $R$ .
- Def. 2.9**      A **set relation** is a relation having only the intrinsic properties of an adicity and simplicity.

All intensional sets are term sets, property sets, or extrinsic property sets; which is to say that no intensional set exists apart from the relation which defines it. No intensional sets are either one-membered or null, because there are no such things as one-membered or null property sets and no monadic or nonadic relations.

As sets, the term set and the property set of a relation  $R$  are pluralities united by their own relation,  $R$ . All other intensional sets are pluralities united by a set relation. Thus in these other cases the application of the function *every* to an intension leads to the emergence of a set relation.

An intensional set is **complete** as well as a unity. The completeness is determined by the function *every*. As well, the

completeness of a term set is the completeness of its terms and the completeness of a property set is the completeness of its properties: if a term set was incomplete the relation could not exist, in which case the term set would not be a term set; and if a property set were incomplete the relation would be a different kind of relation.

Since a relation *R* and its term set are each a unity, it is tempting to identify them and say that a relation *is* the set of its terms. But this is impossible, since set-membership would then be a relation between *R* and each of the terms of *R*; and, also, *R* could not be identical with both its term set and its property set, since this would make these two sets identical. Thus although *R* unites both its term set and its property set, it is identical with neither.

**Def. 2.10**      An **enumeration** of an intensional set is a list of the names or descriptions of every member of that set.

**Kinds** of relations are distinguished by enumeration of their intrinsic property sets; and **instances** of particular kinds of relations are distinguished by enumeration of their term sets, and, if necessary, by enumeration of their extrinsic property sets. (See also Defs. 2.26 and 2.28.)

**Def. 2.11**      The **set-defining rule** of an intensional set is a rule stating the conditions of membership in that set: conditions both sufficient and necessary.

A set-defining rule is thus is a statement of the intension of the set: it states the extrinsic property or properties possessed by all and only the members of the set.

An important intensional set is a similarity set:

**Def. 2.12**      A **similarity set** is the intensional set consisting of every relation having a property set similar to (Def. 2.16) a given property set.

Using the standard symbolism for sets, given a property set  $P$ , its similarity set is  $\{x: XtP\}$  and  $tP$  is its intension — an extrinsic property of each member,  $x$ . Because of the significance of intensions in intensional sets, we adopt an alternative symbolism for  $\{x: XtP\}$ , namely:  $\{A(tP)\}$ . In this symbolism we thus leave out the variable and make explicit both the function *every*, with  $A$ , and the intension; the set is shown, as usual, by the braces. More generally, intensional sets may be represented by  $\{A(RT)\}$ , where  $R$  is a kind of a relation and  $T$  is the kind of one of its terms;  $RT$  is then the intension of the set  $\{A(RT)\}$ , such that every  $RT$  is  $\{A(RT)\}$  and any- $\{A(RT)\}$  is  $RT$ . Still more generally, intensions may be polyadic and they may be combined by *intensional connectives*, to be defined shortly.

We might also define an intensional set by  $\{A(Rt)\}$ : by a particular instance of a relation and a particular instance of one of its terms, rather than by kinds; but there is no use for such intensions in mathematics. In everyday use, an expression such as  $\{x: xRt\}$  might be “Every object in this box”. In such a case the relation  $R$ , or *in*, operates on a basis of kind, or similarity, since each object in the box has its own, similar, instance of the relation *in* to the box. However, the term  $t$ , or box, operates on a basis of identity, since each object is in the one, identical, box. But in mathematics this does not arise: all the features of an intension work on a basis of similarity, not of identity. For example, with a universe of discourse consisting of the natural numbers, in the set  $\{x: x < m\}$ , where  $m$  is a particular natural number, every member of the set has its own relation of *less than*, all similar, and its own instance of  $m$ , all similar. We see this if we begin to enumerate the set in detail, as  $1 < m$ ,  $2 < m$ ,  $3 < m$ , ... Thus, mathematically, in an expression such as  $xRt$ ,  $R$  stands for any instance of  $R$  and  $t$  stands for any instance of  $t$ , which means that  $xRt$  stands for any relation  $x$  related to an instance of  $t$  by an instance of  $R$ , or  $xRT$ .

We will later enlarge the concept of intensional set to include *compound relations* (Def. 4.2), which are unified by relations whose property sets include one or more of the properties of their terms as well as the property set of a set relation; and *wholes* (Def. 4.4), whose property sets include one or more novel properties, as well as the property set of a set relation.

\* \* \*

Certain operations, or functions — necessities — called **connectives** may hold between intensions and between intensional sets, such that the argument of a connective is one or two intensions or sets and the value is another intension or set. We are concerned at present with two kinds:

**Def. 2.13** Those relations between extensions which are defined by means of *identity* are called **extensional connectives**.

**Def. 2.14** Those relations between intensions which are defined by means of *similarity* are called **intensional connectives**.

The distinction between extensional and intensional connectives exists because although it makes sense to speak of particular members, subsets, unions, intersections, etc. of property sets, to do so is of no interest. This is because we are not interested in individual instances of properties, but only in kinds of properties. For this reason intensional connectives are defined by means of similarity, analogously to the definitions of the extensional connectives by means of identity.

We are going to define six extensional connectives and six intensional connectives, and then seek the relations between them. The connectives are represented by the words *equivalent to*, *and*, *or*, *implies*, *difference*, and *not*; we assume that these words have natural, or primitive, meanings, which will be explained in Chapter

3. The first connective to be defined, set identity, is not strictly speaking a connective, although we treat it as one for convenience; that is, it does not connect extensions although it does connect the names of extensions.

**Def. 2.15** Two intensional sets,  $S$  and  $T$ , are **identical**, symbolised by  $S=T$ , if each member of  $S$  is *identical* with a member of  $T$ , and *vice versa*.

$S=T$  means both that for all  $x$ ,  $(x|S)q(x|T)$  and  $(x|T)q(x|S)$ ; or  $(x|S)s(x|T)$ .

Note that what is usually called set equality is here set identity.

**Def. 2.16** Two property sets, or kinds of relation,  $S$  and  $T$ , are **similar**, symbolised by  $S \sim T$ , if each member of  $S$  is *similar* to a member of  $T$ , and *vice versa*.

We can now distinguish two kinds of set-membership:

**Def. 2.17** **Intensional set-membership** is membership in identical sets and **intrinsic property set-membership** is membership in similar sets.

We distinguish the two kinds of membership, by context: in  $x|S$  the membership,  $|$ , is intensional set-membership, and in  $X|S$  it is intrinsic property set-membership; and whenever the word membership is used hereafter, the context will make clear which kind it is.

The membership in an extrinsic property set is intensional set-membership.

So now we may say that if  $S \sim T$  then for all  $X$ ,  $(X|S)q(X|T)$  and  $(X|T)q(X|S)$ , or  $(X|S)s(X|T)$ ; and this is true for every

intrinsic property set that is similar to  $T$ , for every member of  $\{A(tT)\}$ .

Invoking Occam's Razor, we have to say that no two intensional members of any one property set are similar.

We will later define *degrees* of similarity (Def. 5.26) and dissimilarity (Def. 5.27) of kinds of relation.

**Def. 2.18** The **intersection** of two intensional sets,  $S$  and  $T$ , symbolised by  $SfT$ , if it exists, is such that each member of  $SfT$  is *identical* both with a member of  $S$  and with a member of  $T$ . If the intersection of  $S$  and  $T$  does not exist,  $S$  and  $T$  are said to be **disjoint**.

(Note: the null set (Def. 2.37) is not an intensional set.)

**Def. 2.19** The **commonality** of two property sets,  $S$  and  $T$ , symbolised by  $SmT$ , if it exists, is such that each member of  $SmT$  is *similar* both to a member of  $S$  and to a member of  $T$ . If the commonality of  $S$  and  $T$ , other than the two universal intrinsic properties of simplicity and adicity, does not exist,  $S$  and  $T$  are said to be **disparate**.

Thus two disparate relations have no intrinsic properties in common other than the fact of being relations: they are simple and have a plurality of terms.

Of any two single properties  $P$  and  $Q$ , either  $PmQ$  does not exist or else  $(PmQ)tPtQ$ .

Since the definition of commonality allows it to be polyadic we may speak of the commonality of all the members of an intensional set; if  $S$  is an intensional set then the commonality of all of its members will be symbolised by  $MS$ .

Because no two members of any one property set are similar, the commonality of a single property set does not exist.

We will later prove Theorem 1 (6.1): for any intension  $RT$ ,  $(M\{A(RT)\})t(RT)$ . Thus for any intensional set,  $S$ ,  $M$  is the function *any* (Def. 2.5), so that  $MS$  is the intension of  $S$ .

**Def. 2.20** The **union** of two intensional sets,  $S$  and  $T$ , symbolised by  $SgT$ , is such that each member of  $SgT$  is *identical* either with a member of  $S$  or with a member of  $T$ , or both.

**Def. 2.21** The **coupling** of two property sets,  $S$  and  $T$ , symbolised by  $SnT$ , is such that each member of  $SnT$  is *similar* either to a member of  $S$  or to a member of  $T$ , or both.

**Def. 2.22** An intensional set,  $S$  is a **subset** of another intensional set,  $T$ , symbolised  $SiT$ , if each member of  $S$  is *identical* with a member of  $T$ , but not *vice versa*. The inverse of subset is **superset**, symbolised by  $h$ .

$SiT$  means that for all  $x$ ,  $(x|S)q(x|T)$  but  $(x|T)Q(x|S)$ . If  $S$  is either a subset of  $T$ , or identical with  $T$ , this is symbolised by  $SkT$ , and its inverse by  $TjS$ .

**Def. 2.23** A property set,  $S$ , is a **subintension** of another property set,  $T$ , symbolised  $SpT$ , if each member of  $S$  is *similar* to a member of  $T$ , but not *vice versa*. The inverse of subintension is **superintension**, symbolised by  $o$ .

$SpT$  means that for all  $X$ ,  $(X|S)q(X|T)$  but  $(X|T)Q(X|S)$ .

We will generally be more concerned with superintension than with subintension, since superintension will later (Def. 4.7) be shown to be the basis of mathematical inference.

We will later (Theorem 10, (6.10) prove that if  $Av(C_1|C_2|...C_n)$  then it follows that  $C_1 oA$ ,  $C_2 oA$ , ...  $C_n oA$ .

**Def. 2.24** The **set difference** of two intersecting intensional sets,  $S$  and  $T$ , symbolised  $S-T$ , if it exists, is the set consisting of those members of  $S$  which are *not identical* with any member of  $T$ .

**Def. 2.25** The **decoupling** of two property sets,  $S$  and  $T$ , which are not disparate, symbolised  $S-T$ , if it exists, is the set consisting of those members of  $S$  which are *not similar* to any member of  $T$ .

Decoupling is submergence of coupling. If  $Ct(A \cap B)$  then the decoupling of  $A$  from  $C$ ,  $C-A$ , is similar to  $B$ , and the decoupling of  $B$  from  $C$ ,  $C-B$ , is similar to  $A$ :  $(C-A)tB$  and  $(C-B)tA$ ; these decouplings exist only if there exist relations having  $A$ ,  $B$ , and  $C$  as their property sets.

For the next definition we assume the existence of an intensional set,  $U$ , called the universe of discourse, having every relation as its extension.

**Def. 2.26** If  $U$  is the universe of discourse and  $S$  is an intensional set then the **extensional complement** of  $S$ , symbolised  $S'$ , is  $S'=U-S$ .  $S'$  is such that  $S \cap S'=U$  and each member of  $S$  is *not identical* with each member of  $S'$ , and *vice versa*.  $S'$  also reads as non- $S$ .

**Def. 2.27** If  $Av(R|S)$  then  $R$  and  $S$  are **intensional complements** of each other, symbolised with a prime:  $S$  is  $R'$  and  $R$  is  $S'$ ; that is, each member of  $S-A$  is *not similar*, or *dissimilar*, to each member of  $S'-A$ , and *vice versa*.  $S'$  also reads as non- $S$ .

For the disparity of  $S-A$  and  $S'-A$ , see Theorem 10 (6.10), Corol.

As an example of intensional complementarity consider the bipossibility of a natural number being either odd or even. If a natural number is  $N$ , odd is  $O$ , and even is  $E$ , then, assuming these all exist,  $Nv((NnO)|(NnE))$ . (It would be incorrect to write this as  $Nv(O|E)$ , or as we would ordinarily say, “A natural number is either odd or even.” What is properly meant by the ordinary language expression is that a natural number is either an odd natural number or an even natural number.) If now  $U$  is the universe of all natural numbers, we can easily see that  $(MU)tNt((NnO)m(NnE))$ ,  $(NnO)t(NnE)'$ ,  $(NnE)t(NnO)'$ ,  $\{A(NnO)\} = \{A(NnE)\}'$ ,  $\{A(NnE)\} = \{A(NnO)\}'$ , and  $\{A(NnO)\}g\{A(NnE)\} = \{A((NnO)m(NnE))\} = \{A(tN)\}$ . (The last identity is later called a *complete disjunction* (Def. 3.13).)

If  $A$  is the antecedent of a bipossibility relation  $Av(P|P')$ , then as a consequence of Theorem 10 (6.10) we can say that  $At(PmP')$ . From this we can prove Theorem 11, (6.11), the negation theorem:  $\{AP\}' = \{AP'\}$  and  $\{AP'\}' = \{AP\}$ . A clear example of this theorem comes from defining a **dissimilarity set** of a property set  $P$  as  $\{A(TP)\}$ ; although such a set is usually of little interest it is significant here because it is the extensional complement of the similarity set of  $P$ ,  $\{A(tP)\}$ , so that  $\{A(TP)\}g\{A(tP)\} = U$  and  $\{A(TP)\} = \{A(tP)\}'$ . But of any property set  $X$ ,  $Xv((XTP)|(XtP))$ , so that  $(TP)t(tP)'$ . Hence  $\{A(tP)\} = \{A(TP)\}' = \{A(tP)'\}'$ .

It is important that extensional connectives — relations such as union and intersection — are distinguished from intensional connectives — relations such as coupling and commonality — by the fact that the former are defined by identity of members of intensional sets while the latter are defined analogously by similarity of members of property sets. In ordinary language, as well as in mathematics, identity and similarity are

frequently interchanged, as if they were synonyms; to do so is to commit the **identity error**, which is not here tolerable. Identity may be thought of as oneness, whereas similarity requires twoness, since it is a dyadic relation; and when similarity is used transitively, a greater number of terms is required. So in general identity implies unity and similarity implies plurality. We will discover the fundamental relation between identity and similarity shortly, in what is called the equivalence theorem (6.5).

Given a property set  $S$  that belongs to the similarity set of  $R$ , then  $StR$  and  $Sl\{A(tR)\}$ ; but because  $StR$ ,  $tS$  is equally an intension of  $\{A(tR)\}$  — that is,  $\{A(tS)\}=\{A(tR)\}$  — and so  $\{A(tR)\}$  has as many instances of its intension as it has members; but we are not distinguishing instances of property sets, only kinds, and so there is only one kind of intension.

**Def. 2.28**  $Y$  is an **instance** of a kind of relation,  $R$ , if and only if  $y$  is a member of the similarity set determined by  $R$ :  $yl\{x: XtR\}$ , or  $yl\{A(tR)\}$ .

Because an instance of  $R$  determines the similarity set of  $R$ , just as  $R$  does, the difference between a kind of a relation and an instance of that relation is often of small account. As we have seen, an instance of a relation is determined by its term set and the kind of a relation is its property set. The instance is a member of the similarity set and the property set is the distinguishing part of the intension of the similarity set; these are symbolised by  $R$  or  $r$ , for the instance, and by  $R$  for the kind, and since the difference between instance and kind is generally trivial it is not usually of great moment which of the symbols  $r$  or  $R$  are used. Thus ‘2’ stands equally for *the* number two — the kind — and for an instance of it. The expression *any instance* is synonymous with *kind*, as is clear from the definition of the function *any*, (Def. 2.5). None the less, there are some differences, as is shown by the intensional sets  $\{A(Rt)\}$  and  $\{A(RT)\}$ .

**Def. 2.29** The **superintension set** of a property set,  $P$ , is that intensional set, each member of which is a superintension of the property set  $P$ :  $\{x: X \circ P\}$ , or  $\{A(\circ P)\}$ . The **subintension set** of a property set  $P$  is  $\{A(pP)\}$ .

We will not generally be much concerned with subintension sets: they are defined here only for the sake of completeness.

**Def. 2.30** If a relation  $R$  has a property set  $R$  then  $R$  is a **representative instance** of a property set  $S$  if and only if  $R$  is a member of the superintension set determined by  $S$ :  $R \in \{X: X \circ S\}$ , or  $R \in \{A(\circ S)\}$ .

**Def. 2.31** A relation  $R$ , of property set  $R$ , is said to **possess** a property, or property set,  $P$ , if and only if either  $R \circ P$  or  $R \circ P$ .

We use representative instances in place of regular instances when dealing with any commonalities of property sets, or couplings thereof, which do not have instances. If relations  $R$  and  $S$  have property sets  $R$  and  $S$  but there does not exist any relation whose property set is  $R \circ S$  then there exists no instance of  $R \circ S$ ; but  $R$  is a representative instance of  $R \circ S$  because  $R \circ (R \circ S)$ ; so also is  $S$  a representative instance of  $R \circ S$ , as is any other superintension of  $R \circ S$ . This is, of course, an ancient usage: Euclid allowed a particular triangle  $ABC$  to represent any triangle, such that only the property of triangularity possessed by  $ABC$  was allowed into consideration.

\* \* \*

It is a fact that every intension is extrinsic to the set that it specifies, and also extrinsic to each member of that set. For

example, in the similarity set {AtS} the intension is tS and this is neither a member of the set nor an intrinsic property of any one of the members. However, some intensions specify by means of intrinsic properties and some by means of extrinsic properties, and we will use that fact to distinguish intrinsic and extrinsic sets.

**Def. 2.32**      An **intrinsic intension** is an intension that specifies a property set that is intrinsic to each member of its intensional set, and that set is called an **intrinsic set**.

**Def. 2.33**      An **extrinsic intension** is an intension that specifies a property set that is extrinsic to each member of its intensional set, and that set is called an **extrinsic set**.

Intrinsic property sets, similarity sets, superintension sets, and subintension sets are intrinsic sets; all other sets are extrinsic sets. The significance of intrinsic sets is that a similarity set and a superintension set represent every instance and every representative instance of a property set.

Because ordinary language presupposes it, we will adopt the convention, when there is no possibility of confusion, that an intension written as only a property set, P, is the intension of an intrinsic set: {AP} represents either {A(tP)} or {A(oP)}. In other words {AP} refers to all instances of P, whether they are instances or representative instances.

And also when there is no possibility of confusion an extrinsic intension such as RT will be written as a single property set. Thus {AS} might represent {ART}. Both these conventions are adopted for the sake of greater generality of statements.

It is not possible for an extrinsic set to be one-membered, as in the open interval (1,3) in the natural numbers, because every intensional set is unified by a set relation and the least adicity of a relation is two. Such a one-membered interval is a *contingent set*,

to be defined later (Def. 2.34); and a null interval such as (1,2) is a *null set* (Def. 2.38).

\* \* \*

In Chapter 6 we will prove the following five theorems that state relations between intensional and extensional connectives, provided that their terms exist.

**The negation theorem (6.11):**  $\{AA'\} = \{AA\}'$  and  $\{AA\} = \{AA'\}'$

**The conjunction theorem (6.7):**  $\{A(AnB)\} = \{AA\}f\{AB\}$

**The disjunction theorem (6.9):**  $\{A(AmB)\} j \{AA\}g\{AB\}$

**The implication theorem (6.6):**  $(A \circ B) s (\{AA\}i\{AB\})$

**The equivalence theorem (6.5):**  $(AtB) s (\{AA\}=\{AB\})$

If  $A \circ B$  we may say that A is larger than B, and B is smaller than A; and similarly if  $\{AA\}i\{AB\}$  we may say that  $\{AA\}$  is smaller than  $\{AB\}$  and  $\{AB\}$  is larger than  $\{AA\}$ . What we are really doing is anticipating later developments by assuming that intensions and extensions, as sets, have natural numbers of members, and these numbers may be compared. So these theorems imply that the larger an intension the smaller its extension: the size of an intension is inversely proportional to the size of its extension. This means that sets determined by simple rules have very large extensions, while small extensions must be determined by complex rules. This latter may seem implausible at first, as shown by the example of “Everything in this box”, which is a small extension determined by a simple extrinsic intension; but “in this box” is a simplification of the intension, since everything in this box is also in this room, in this building, in this town, on this planet, in this

solar system, etc. Mathematically this situation occurs with least upper bounds and greatest lower bounds among natural numbers. An interval specified by a g.l.b. and an l.u.b. properly includes in its specification all of its lower bounds and all of its upper bounds: to specify the greatest or least is to presuppose the existence of an ordered set, which set is part of the intension. So the intension is large while the extension is small — compared with, say, the set of natural numbers, which has a small intension (see Chapter 5) and a large extension.

An intensional set may in principle be specified either by a rule or by an enumeration, but an enumeration is not a set-defining rule both because it does not specify an intension, and because an enumeration is in one-one correspondence with the extension it specifies, so that the larger the enumeration the larger the extension, as opposed to the principle that the larger the intension the smaller the extension. Notice that it is possible for a set-defining rule to be disguised as an enumeration, as in the supposed enumeration of the natural numbers,  $\{1, 2, 3, \dots\}$ ; the ellipsis, standing for “and so on” contains a rule of succession and  $\{1, 2, 3, \dots\} = \{x: xS1\} = \{A(S1)\}$ , where  $S$  is the transitive relation of successor.

We will later prove the theorem (6.3):

**Theorem 3:** A set  $S$  is an intensional set if and only if both  $MS$  exists and  $S = \{A(MS)\}$ .

The truth of this theorem is also clear from the fact that an intension of a set is defined as that property set possessed by all and only the members of that set: which is the set of everything that possesses the commonality of that set. We say that  $MS$  does not exist if at least two members of the intensional set are disparate: they have no more than simplicity and adicity in common.

Intensional sets are so named because they have intensions. Because every intensional set has its membership necessitated by its intension and because every intensional set is complete, every intensional set may also be called a **necessary set** and a **complete set**. This point is made because the concept of set may be enlarged to include sets which are not intensional sets and are contingent and incomplete. An intensional set was characterised by it consisting of (i) a plurality of members which is (ii) defined by an intension and (iii) unified by a relation, such as a set relation. We extend the concept of set to a plurality of members which is not unified: a concept of a set which is not a term set of a relation. This is plausible because linguistically there is no difference between a many membered set and a plurality: each is many yet spoken of as one.

We saw that if  $A$  and  $B$  are intensional sets and  $A \models B$ , then  $(x \models A) \supset (x \models B)$ ; but the converse does not hold: if  $(x \models A) \supset (x \models B)$  and  $B$  is an intensional set, then we can say that  $A \models B$ , but it does not follow that  $A$  is an intensional set. We see this when  $B$  is an intensional set and  $A$  is a random selection (Def. 3.6) of members of  $B$  and so has no intension:  $A$  can only be enumerated.  $A$  is a plurality because  $A$  is a subset of  $B$ ,  $A \models B$ , since  $(x \models A) \supset (x \models B)$ ; but because  $A$  has no intension and is not defined by the function *every*, it is incomplete, it is a plurality without unity, and its membership is contingent, so it cannot be specified by a set-defining rule, although it can be specified by an enumeration. Because standard usage calls this  $A$  a set rather than a mere plurality, we will continue to so use the word set, by defining an incomplete or contingent set:

**Def. 2.34** An **incomplete, or contingent, set** is any enumerable plurality of members which has no intension and no unifying relation; that is, any extension which can be

enumerated but which cannot be specified by a rule,  
because it is not a value of the function *every*.

We enlarge the symbolism of italic upper-case letters, to include contingent sets; and of *l*, to include membership in contingent sets. And we also allow one-membered contingent sets, because the one member may be enumerated — even though it is not a plurality; but we will include one-membered sets in the concept of plurality, for simplicity.

The commonality of a one-membered set is the property set of that one member.

The definitions above of the extensional connectives — set identity, intersection, union, subset, set difference, and complement all apply to incomplete sets. It is possible but unlikely that a subset of an incomplete set, or a union, intersection, or difference of any two of them, may be complete sets. Obviously, the definitions of the intensional connectives cannot be applied to incomplete sets.

We will later prove the theorem (6.4):

**Theorem 4:** A set  $S$  is a contingent set if and only if it has at least one member and either its commonality,  $MS$ , does not exist or else  $MS$  exists and  $Si\{A(MS)\}$ .

If  $S$  is a contingent set then  $MS$  is not identical with the function *any*  $S$ . Consequently we define:

**Def 2.35** Extensional *any* is the use of the word *any* applied to a contingent set,  $S$ , meaning a random (Def. 3.6) selection of a member of  $S$ .

Thus *extensional any* is the extensional meaning (Def. 3.2) of the word *any*.

An incomplete set  $S$  is incomplete and contingent because it cannot be determined by  $MS$ , so cannot be determined by a rule, so can only be determined by enumeration. Suppose that  $M\{4,5,9\}$  is *natural number less than 10*, or  $N$ , say, and  $\{AN\}$  is the set of every natural number less than 10; thus  $\{4,5,9\}i\{AN\}$ , so  $\{4,5,9\}i\{A(M\{4,5,9\})\}$  and is thereby incomplete. Also, for any set  $S$  there is a plurality of subsets of  $\{AMS\}$ , all of which have the same commonality,  $MS$ ; there is thus no intensional function (Def. 3.7) from  $MS$  to any of these subsets, and so each subset is one of a plurality of possibilities, given  $MS$ , and so contingent. In the example, there is no intensional function from  $N$  to  $\{4,5,9\}$ , and  $Nt(M\{4,5,9\})t(M\{1,6,7,9\})$ . Also, if  $MS$  does not exist, because at least two members of  $S$  are disparate,  $S$  cannot be an intensional set, by Theorem 3 (6.3), and so must be an incomplete set. An incomplete set is not a unity, except nominally, because it does not have a set relation to unify it.

Notice that the disjunction theorem, (6.9),  $\{A(AmB)\}j\{AA\}g\{AB\}$ , proves that there is no closure on union of intensional sets: the union of two intensional sets is not necessarily an intensional set, since it may be a subset of an intensional set and so may be a contingent set. This is further discussed in Chapter 3, in connection with the truth tables for disjunction.

**Def. 2.36** An **extensional set**, or **extension**, is any set which is either a complete set or an incomplete set.

**Def. 2.37** A contingent set is **extensionally complete** in the sense that its extension is in one-to-one correspondence with its enumeration.

Thus a contingent set is intensionally incomplete but extensionally complete; these are respectively the intensional and extensional meanings (Defs. 3.1, 3.2) of the word *complete*.

\* \* \*

The concept of set may be extended further in that the intersection of two disjoint sets may be called a set. Such a set obviously has no members and so may be described as a set without an extension. In the present context the definition is self-contradictory, since a disjoint intersection is a non-intersecting intersection, or an extension without an extension. So such a set does not exist except nominally.

**Def. 2.38**      **A null set is a set which has no extension.**

A null set is here symbolised, as usual, by  $\phi$ . Note that the difference between the expressions “A null set” and “The null set” is only nominal.

**Def. 2.39**      **A nominal set is a set which is either an extensional set or a null set.**

The definitions of the extensional connectives may be extended to null sets, analogously to our earlier definitions. As relations, these connectives do not exist when one or more of their terms are null sets, since null sets do not exist and a relation cannot exist unless all of its terms exist. In such cases they are purely nominal relations (Def 1.7). Also, the word *every* may be used to define a null set, as with the set of every even prime number greater than two, but in such cases the function *every* does not exist: the *every* is a purely nominal relation.

\* \* \*

We can now distinguish three distinct set theories.

**Def. 2.40**      **Intensional set theory** deals only with intensional sets, also called natural sets, complete sets, and necessary sets.

**Def. 2.41**      **Extensional set theory** deals with extensional sets: sets which are either complete sets or incomplete sets, without distinction.

**Def. 2.42**      **Nominal set theory** deals with nominal sets: sets which are either extensional sets or null sets.

We can characterise the three kinds of sets very simply: an intensional set has both an intension and an extension; an exclusively extensional set (Def. 3.2) has an extension but no intension; and an exclusively nominal set (Def. 3.3) has neither intension nor extension.

The content of intensional set theory is a subset of the content of extensional set theory, which is a subset of the content of nominal set theory. So the existence of sets of intensional set theory is a sufficient condition for the existence of sets of extensional set theory, which in turn is a sufficient condition for the existence of sets of nominal set theory; but the existence of sets of nominal set theory is only a necessary condition for sets of extensional set theory, which in turn is only a necessary condition for sets of intensional set theory. And nominal set theory has the greatest generality, while intensional set theory has the least.

We note that the relations between extensional sets are purely nominal relations, since pluralities cannot be single terms of a relation. Thus it is not possible to have an intensional theory of extensional sets. Equally the relations between null sets and other sets — such as  $\phi \models S$  and  $U' = \phi$  — are purely nominal relations because no relation can exist if one or more of its terms do not exist. Thus there is no intensional theory of extensional set or of nominal sets.

### 3. Three Kinds of Mathematical Meaning

With three kinds of set theory we can distinguish three kinds of mathematical meaning: intensional, extensional, and nominal.

**Def. 3.1** A mathematical symbol, name, or description has **intensional meaning** if its meaning is a relation, or one or more intrinsic or extrinsic properties of a relation.

**Def. 3.2** A mathematical symbol, name, or description has **extensional meaning** if its meaning is an extension; and it has **exclusively extensional meaning** if it has extensional meaning but no intensional meaning.

Not every intensional meaning is an intension, but all intensional meanings can generate a variety of intensions. For example, if a relation  $R$  has property set  $R$ , then  $tR$ ,  $oR$ , and  $RT$  are intensions. However, every intension defines an extension so intensional meaning is a sufficient condition for extensional meaning; but extensional meaning is only a necessary condition for intensional meaning since not every extension has an intension.

**Def. 3.3** A mathematical symbol, name, or description has **nominal meaning** if its meaning is a nominal set; and it has **exclusively, or purely, nominal meaning** if it has no extensional meaning.

Clearly, extensional meaning is a sufficient condition for nominal meaning, but nominal meaning is only a necessary condition for extensional meaning.

**Def. 3.4** A mathematical symbol, name, or description has **purely intensional meaning** if it has intensional meaning, no

exclusively extensional meaning, and no exclusively nominal meaning.

**Def. 3.5** A mathematical symbol, name, or description has **purely extensional meaning** if it has extensional meaning, no intensional meaning, and no purely nominal meaning.

Purely intensional meanings entail some extensional meaning, and some nominal meaning, since every intension defines an extension and every extension may be described or named; but they entail no more extensional and nominal meanings than this minimum. Similarly, purely extensional meanings entail a minimum of nominal meaning. For brevity, this relationship will be expressed as IqEqN, where I, E, and N stand for intensional, extensional, and nominal meaning.

Since some nominal sets have neither intensions nor extensions, their nominal meaning is meaning by verbal analogy to intensional or extensional meaning. For example, given that the words *triangle*, *equilateral*, and *right angle* have intensional meaning, these meanings may be coupled to give intensional meaning to *equilateral triangle* and *right angled triangle*; such coupling is indicated grammatically by verbal succession. So by verbal analogy these words also may be conjoined to give *right angled equilateral triangle*, which has neither intensional meaning nor extensional meaning, but does have nominal meaning, by analogy. Or we might have an enumeration of a supposed extensional set, such that every item in the enumeration has only nominal meaning; so by verbal analogy this enumeration has nominal meaning but no extensional meaning.

We have already made the claim, on the basis of Occam's Razor, that certain names of relations have purely nominal meaning: namely, the supposed relation *term of*, and supposed monadic relations such as self-similarity, improper part, and improper subset, because they are relations which multiply

extravagantly beyond necessity. They were defined as purely nominal relations (Def. 1.7): relations which do not exist except nominally; that is, except in language.

We could say that purely nominal relations exist only in language, as names or descriptions of relations; but this can be misleading because there are also genuine relations in language, in the form of relations between words: succession, and grammatical relations. When these grammatical relations relate words that have nominal meaning only, their grammatical meaning is meaning by analogy with grammatical relations between words that have intensional or extensional meaning. For example, we will later (see Chapter 5) argue that there are intensional numbers — that is, numbers having intensional meaning — but that the number zero has only nominal meaning. If, now,  $a$  and  $b$  are intensional numbers then  $a+b$  is intensionally defined, but  $a+0$  and  $b+0$  are not defined because addition, as a relation, cannot exist if one of its terms does not exist. But  $a+0$  and  $b+0$ , as symbolic expressions, have meaning by analogy with  $a+b$ , and this meaning by analogy is exclusively nominal meaning.

These three concepts of intensional, extensional, and nominal meaning are themselves analysable into Frege's sense and reference. There are three kinds of sense: intensional sense, or ideal intensional meanings; extensional sense, or ideal extensions; and nominal sense, or sense by grammatical analogy with statements of intensional or extensional sense. And there are two kinds of reference: intensional reference, or real relations or relational properties; and extensional reference, or real extensions. Thus exclusively nominal meaning has sense but no reference. So the phrases 'right angled triangle' and 'right angled equilateral triangle' both have nominal meaning, but the second has only nominal meaning, while the first also has intensional and extensional meaning, as well as intensional and extensional reference.

Since the set of all intensional meanings is correlated with a subset of the set of all extensional meanings, and the set of all extensional meanings is correlated with a subset of the set of all nominal meanings, intensional meaning has the least generality and nominal meaning has the most: **IqEqN**.

However, intensional meanings have least arbitrariness, and nominal meanings have most. Arbitrariness in intensional meaning is confined to arbitrary manipulations of intensions; the resulting sets, if they exist, are necessary sets: their membership is necessitated by the intension, and thereby unarbitrary.

With exclusively extensional meaning — contingent sets — far more arbitrariness is possible: most random sequences of natural numbers are intensionless extensions, or purely extensional sets, and most random selections of referring proper names or definite descriptions are enumerations of purely extensional sets. Because of this we may define:

**Def. 3.6**      The **random** or the **arbitrary** is some degree of contingency (Def **1.16**).

Thus random membership in a set has exclusively extensional meaning — that is, no intensional meaning. This is because necessary membership is membership in a necessary set and contingent membership is membership in a contingent set.

With exclusively nominal meaning still more arbitrariness is made possible with ridiculous combinations of symbols, words, and descriptions, as with the even prime numbers greater than two, the rational square root of two, square-circle, the nearly infinite, military intelligence, honest politician, and truth in advertising.

Two reasons, more important than arbitrariness, for distinguishing these three kinds of meaning are that axiom generosity is possible only with purely intensional meanings, and paradox is possible only with exclusively nominal meanings.

Axiom generosity, which is the cornucopia of definitions and theorems that a good axiom set pours out, is possible only with intensional meaning: since intensional meanings are relations or relational properties, axiom generosity results from cascading emergence. Only relations can produce cascading emergence because only relations both have terms and can be terms; so only intensional meanings can produce cascading emergence. Paradox and contradiction cannot exist in reality so they are possible only in language: that is, language that has only nominal meaning. In language we can name the unnameable, describe the indescribable, refer to the unreferable, classify things as unclassified, and specify any contradiction we please; but we cannot do such things outside of language<sup>3</sup>.

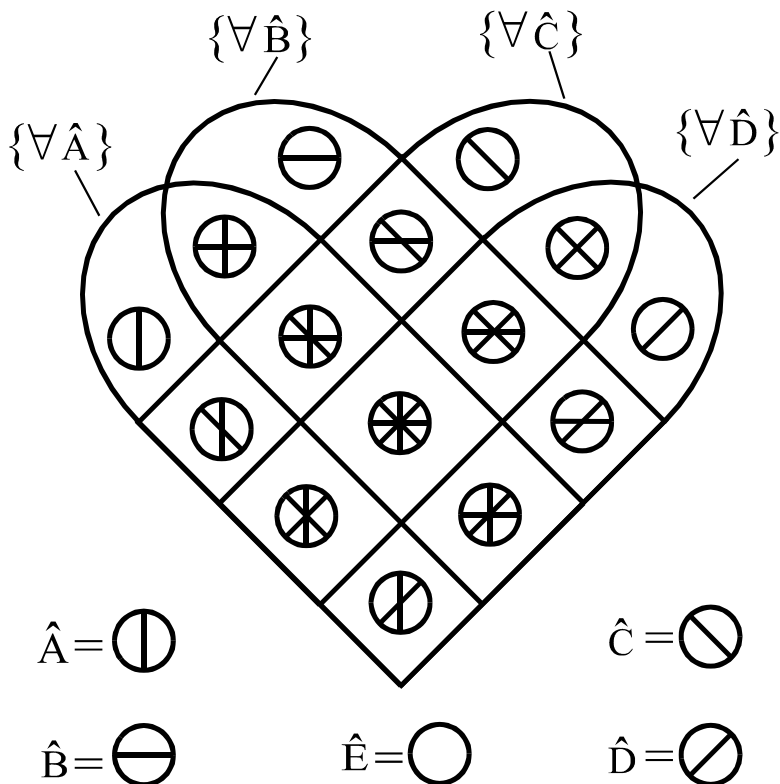
Everyday mathematics is a mixture of all three kinds of meaning. For example, whenever mathematicians define a set with a genuine set-defining rule, they have intensional meaning; when they define by means of intensionless extensions, as with enumerated sets and enumerated functions, they have extensional meaning but usually no intensional meaning; and whenever their definitions lead to paradox, as with the definition of the set of all sets, or the set of all sets which are not self-membered (see below), they have some purely nominal meaning.

\* \* \*

Intensional meanings may be represented graphically, by **pattern diagrams**, in a manner complementary to the graphical

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<sup>3</sup> The so-called optical illusions — two-dimensional drawings of impossible three-dimensional objects — are not impossibilities in the current sense: they are perfectly possible two-dimensional drawings.



*Fig.3.1*

representation of extensional meanings by Venn diagrams. The use of patterns is natural enough, since patterns are relations; parts of the patterns represent intrinsic properties of the relations. This is shown in Fig. 3.1, in which four intensions, A, B, C, and D are represented by four patterns; their sets together form a quadruple Venn diagram, each set being shaped like an eight-paned arched window. The commonality of the four intensions,  $M(A,B,C,D)$ , is

represented by E, an empty circle; and we assume that  $\{AE\}$  is identical with the quadruple Venn. Four of the theorems relating the intensional and extensional connectives are illustrated in the diagram: conjunction,  $\{A(AnB)\} = \{AA\}f\{AB\}$ ; both kinds of disjunction,  $\{A(AmB)\} h \{AA\}g\{AB\}$  — incomplete (Def. 3.14) because  $(AmB)tE$  and  $\{AA\}g\{AB\} i \{AE\}$  — and complete (Def. 3.13) because  $\{A(AnBnCnD)\} = \{AA\}f\{AB\}f\{AC\}f\{AD\}$ ; implication,  $(AoE)$  and  $(\{AA\}i\{AE\})$ ; and equivalence,  $((AmC)t(BmD))$  and  $(\{A(AmC)\}=\{A(BmD)\})$  — because  $(AmC)t(BmD)te$  and  $\{A(AmC)\}=\{A(BmD)\}=\{Ae\}$ .

\* \* \*

We have already distinguished the intensional and extensional meanings of *any* and *complete*; we see the nominal meanings of these if we speak of any member of the null set, or of the completeness of the null set. The importance of distinguishing these three kinds of mathematical meaning makes some further detailed examples of them worthwhile.

The differences between the intensional, extensional, and nominal meanings of the word *necessity* are a good illustration of the three kinds of meaning. Intensional necessity is either the relation of singular possibility or else an extrinsic property. Extensional necessity is universality, as occurs with subsets: if the contingent set  $A$  is a subset of the contingent set  $B$ , then members of  $A$  are universally members of  $B$ , but only contingently so, hence not intensionally necessarily so. Nominal necessity is the necessity of truth-functional tautology, which requires such strange things as the paradoxes of material implication, so that any contradiction nominally necessitates any other proposition, and a tautology is nominally necessitated by any proposition. Indeed, these paradoxes

show that a formal logic based on Boolean algebra cannot properly represent intensional, or rational, thought.

The difference between singular possibility and universality is that it is inconceivable for singular possibility to be otherwise, but not for universality to be otherwise: we cannot conceive of  $2+3=5$  being otherwise<sup>4</sup> but we can conceive of a particular social gathering having a different guest list. Again, of the seven examples of exclusively nominal meaning above, the first four are necessarily contradictions and the last three are probably universally contradictions. Observe that singular possibility is a sufficient condition for universality, which in turn is a sufficient condition for tautology, but tautology is only a necessary condition for universality, which in turn is only a necessary condition for singular possibility: **IqEqN**.

The three kinds of necessity appear in the three meanings of the word *function*. We have been using the concept of function since the relation of necessity was introduced, but we can now be somewhat more precise about it, so as to distinguish intensional functions from extensional and nominal functions.

**Def. 3.7**      An **intensional function** is any relation whose property set is a superintension of the property set of the necessity relation, of singular possibility. Each of its possible antecedents is called an **argument** of it, and its singular

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<sup>4</sup>We can say, or write, otherwise, as in  $2+3=7$ , but such statements have nominal meaning only. Each part of this statement, such as 2, +, =, has intensional meaning, as does each side of the equation, but the equation as a whole does not. What the equation as a whole states is impossible and therefore has nominal meaning only. To conceive is to have intensional or extensional meaning in the mind; while to “think” using silent speech that has exclusively nominal meaning is not to conceive (see Def. **13.47**).

consequent, for each argument, is called the **value**, or **image**, of that argument. The set of every possible argument of it is called its **domain** and the set of every possible value of it is called its **co-domain** or **range**.

Like all relations, instances of functions are determined by their terms — argument and value — and kinds of functions are determined by their property sets. But the kind of relation called an intensional function is determined by the commonality of the property sets of all intensional functions, which is the property set of necessity, which is the property set of singular possibility.

Intensional functions relating real relations are **causations**.

**Def. 3.8**      A **contingent function** is a contingent set of assignments of a single value to every member in an extensional set called the **domain** of the function, which set is the extensional set of its arguments, the values being the members of an extensional set called the **co-domain** or **range** of the function.

We note that an assignment is a relation, usually created by a mind.

**Def. 3.9**      An **extensional function** is a set of ordered pairs determined by either an intensional function or a contingent function, where each pair is composed of first an argument of the function and second its corresponding value. The set of ordered pairs must be intensionally complete if it is determined by an intensional function, and it must be extensionally complete if it is determined by an extensional function.

- Def. 3.10**      An **exclusively extensional function** is an extensional function determined by a contingent function.
- Def. 3.11**      A **nominal function** is an extensional function whose domain, range, or both, are nominal sets.
- Def. 3.12**      An **exclusively nominal function** is a nominal function whose domain, range, or both, are null.

A contingent function may be enumerated but cannot be specified by a rule. A contingent function universally assigns unique values to its arguments, as does an intensional function, but this universality does not arise from an intensional necessity. So the existence of an intensional function is a sufficient condition for the existence of an extensional function, which in turn is a sufficient condition for a nominal function, but the converses are only necessary conditions: **IqEqN**. The intensional function is a necessity and the extensional function is a universality. Also, an extensional function may be far more arbitrary than an intensional one, while an exclusively nominal function may be arbitrary beyond belief. As a relation, an exclusively nominal function does not exist because one or both of its terms do not exist; it is thus a purely nominal relation.

Other clear examples of the three kinds of meanings are provided by the truth-functional sentential connectives (Table 3.1 below): negation, conjunction, disjunction, implication, and equivalence. If A and B are intensions then the intensional meanings of the sentential connectives are the intensional connectives, assuming that they exist:

$A'$  means “Non- $A$ ” or “Not  $A$ ” (3.1)

$A \wedge B$  means “ $A$  and  $B$ ” (3.2)

$A \vee B$  means “ $A$  or  $B$ ” (3.3)

$A \supset B$  means “If  $A$  then  $B$ ” or “ $A$  implies  $B$ ” (3.4)

$A \equiv B$  means “ $A$  is equivalent to  $B$ ” (3.5)

If  $A$  and  $B$  are extensions then the extensional meanings of the sentential connectives are the extensional connectives:

$xlA'$  means “ $xlA$ ” or “Not  $xlA$ ” (3.6)

$xl(A \wedge B)$  means “ $xlA$  and  $xlB$ ” (3.7)

$xl(A \vee B)$  means “ $xlA$  or  $xlB$ ” (3.8)

$AiB$  means “ $xlA$  implies  $xlB$ ” (3.9)

$A=B$  means “ $xlA$  is equivalent to  $xlB$ ” (3.10)

The nominal meanings of the sentential connectives are the standard truth-functional connectives, defined by truth tables: negation ( $U$ ), conjunction ( $z$ ), disjunction ( $y$ ), implication ( $h$ ), and equivalence ( $1$ ), as in the standard Table 3.1. These are part of a Boolean algebra, which has intensional meaning, but the application of this Boolean algebra to logic has nominal meaning only because of paradox.

P	Q	UP	PzQ	PyQ	PhQ	P1Q
T	T	F	T	T	T	T
F	T	T	F	T	T	F
T	F		F	T	F	F
F	F		F	F	T	T

Table 3.1.

When we defined the extensional connectives and the intensional connectives, using identity and similarity, we did so by means of words such as *not*, *and*, *or*, and *if...then...* which suggests circularity in the definitions. In fact, we all have meanings for these words, which we learn soon after we learn to talk. It is these natural meanings that were intended in the definitions of the intensional and extensional connectives, and it is now proposed that these natural meanings are the extensional meanings; the intensional and nominal meanings are then derivative from the natural meanings. So properly speaking, the extensional connectives are primitive: the supposed definitions of them given above are characterisations in terms of our natural meanings. The nominal meanings of the sentential connectives are familiar enough, but a little must be said about the intensional sentential connectives, and, in the next chapter, about extensional implication.

Intensional negation occurs only with intensional complements (Def. 2.27), such as *inside* and *outside*, *true* and *false*, *on* and *off*, and *similar* and *dissimilar*. Any other use of negation in intensional discourse is metalinguistic, as in the correction of an error, the disproof of a conjecture, or a proof of non-existence. A discourse in which everything has purely intensional meaning states only intensional fact — real or ideal — and so has no use for negation as a metalinguistic operation. So for most relations there are only nominal complements: the nominal complement of the relation being its absence, or non-existence, which has no intensional meaning. So there is no closure on intensional negation: very few intensional meanings have intensional complements and so may be negated intensionally, although all of them may be negated nominally.

When a relation has a symbol of its own, this non-existence, or nominal meaning, is shown as usual by a vertical stroke through the symbol. Thus coupling is  $n$  and its absence is  $N$ , necessity is  $q$  and its absence is  $Q$ , and intersection is  $f$  and its absence, or disjointness, is  $F$ .

There are, of course, many words which have nominal meaning only and which, although of utility in everyday discourse, have no place in purely intensional discourse: words such as *impossible*, *non-existent*, and *unthinkable*.

Intensional conjunction may be coupling of abstract ideas, as in *right triangle*, or coupling of propositions — which are structures of abstract ideas. Coupling of ideas is not always possible; when not, the extensions of the ideas are disjoint. When this happens, as in *square circle*, the words for the ideas may be conjoined but the result has exclusively nominal meaning; so there is no closure on intensional conjunction. Coupling of propositions is usually implicit, linguistically: the relation that couples them is succession — sentential succession, and hence propositional succession. That is, the conjunction is nominally implicit but intensionally explicit, in that there is no *and* or *but*, but the linguistic relations of succession are genuine. For clarity, the ampersand symbol, &, will also be used here for intensional conjunction, particularly in the case of intensional propositions.

Intensional disjunction — commonality — also has no closure, for two reasons: the commonality of two intensions may not exist — the intensions are disparate — and, secondly, if it does exist the union of their extensions may a subset of an intensional set and so itself may not be an intensional set, as shown by the disjunction theorem (6.9):  $\{A(A \text{ and } B)\} \text{ } j \text{ } (\{A\} g \{B\})$ . As a result of this second reason, there are two possible kinds of disjunction of intensions: that in which the resulting extension is an intensional, or complete, set, and that in which it is not an intensional set, hence an incomplete set.

**Def. 3.13**      **A complete disjunction** is an intensional commonality, or disjunction, whose extension is a complete, or necessary, set.

**Def. 3.14** An **incomplete disjunction** is an intensional commonality, or disjunction, whose extension is an incomplete, or contingent, set.

We saw (Theorems 3 (6.3) and 4 (6.4)) that an extension  $S$  is complete if and only if  $MS$  exists and  $S = \{AMS\}$ , while  $S$  is incomplete if either  $MS$  does not exist or else  $S \neq \{AMS\}$ . So if  $\{AA\}g\{AB\} = \{A(AmB)\}$  then the intensional disjunction  $A \cap B$  is a complete disjunction; and if  $\{AA\}g\{AB\} \neq \{A(AmB)\}$  then the intensional disjunction  $A \cap B$  is an incomplete disjunction. (If  $M(\{AA\}g\{AB\})$  does not exist it is because  $A$  and  $B$  are disparate, which means that  $A \cap B$  does not exist; in this case there is no intensional disjunction.)

An example of each kind of disjunction, shown in Fig. 3.2, will clarify this point. First, take our earlier example in which the intensions of *natural number*, *odd*, and *even* were  $N$ ,  $O$ , and  $E$ . Then the intensions of the sets of the odd numbers and of the even numbers are  $N \cap O$  and  $N \cap E$  and the extensional disjunction of these sets is  $\{A(N \cap O)\}g\{A(N \cap E)\}$ , which, as we know, is the set  $\{AN\}$ . Intensionally, the disjunction is  $(N \cap O)m(N \cap E)$ , which is similar to  $N$ :  $(N \cap O)m(N \cap E) \leq N$ . So it follows, by the equivalence theorem (6.5), that  $\{A((N \cap O)m(N \cap E))\} = \{A(N \cap O)\}g\{A(N \cap E)\}$ . So intensionally what is either an odd number or an even number is a number; the disjunction is complete and conforms with normal usage.

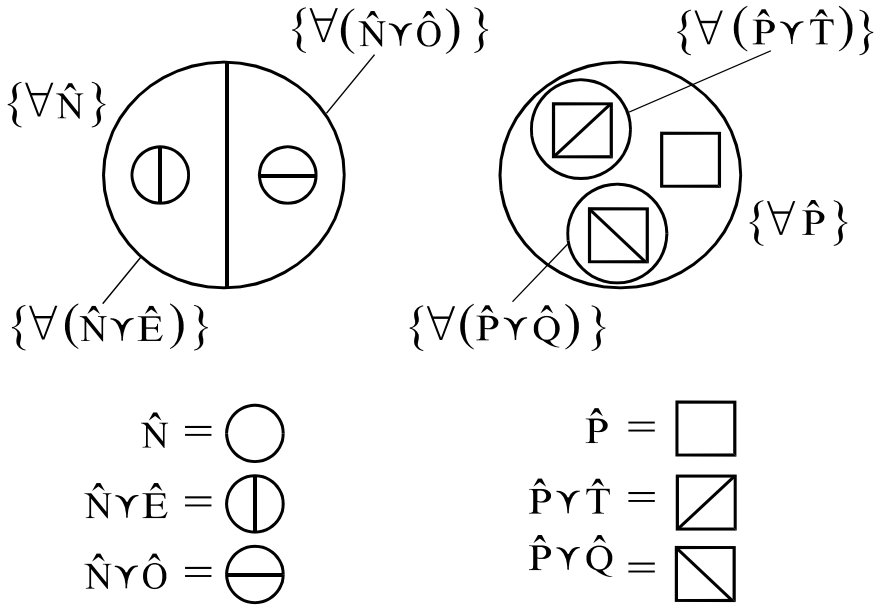


Fig. 3.2

For the second example, let the intensional meanings of *simple polygon*, *trilateral*, and *quadrilateral* be  $P$ ,  $T$ , and  $Q$ . Then  $TnP$  and  $QnP$  are the intensional meanings of *triangle* and *quadrangle*, and  $\{A(TnP)\}$  and  $\{A(QnP)\}$  are the intensional sets of every triangle and of every quadrangle. So  $\{A(TnP)\}g\{A(QnP)\}$  is the extensional disjunction of these disjoint sets; anything belonging to this set is, extensionally, either a triangle or a quadrangle. However the intensional disjunction of  $TnP$  and  $QnP$  is  $(TnP)m(QnP)$ , and  $((TnP)m(QnP))tP$ : the commonality of  $TnP$  and  $QnP$  is  $P$ . So anything which is a member of  $\{A((TnP)m(QnP))\}$  is identically a member of  $\{AP\}$ . Hence, intensionally, anything which is either a triangle or a quadrilateral is, more accurately, either a triangle or a quadrilateral or any other polygon. So  $\{A((TnP)m(QnP))\}h(\{A(TnP)\}g\{A(QnP)\})$ . In

this case intensional disjunction is peculiar: it is one or the other or neither, where the *neither* refers to the set difference between the intensional and the extensional sets:

$\{A((TnP)m(QnP))\} - (\{A(TnP)\}g\{A(QnP)\})$ . This is the kind of intensional disjunction that is incomplete: it is incomplete because the corresponding union is a contingent set, an incomplete set, hence the name incomplete disjunction.

So, more generally,  $AmB$  means “A or B” if  $AmB$  is complete, and it means “A or B or neither” if  $AmB$  is incomplete. In the incomplete case the *neither* refers to any member of  $\{A(AmB)\}$  which is neither an A nor a B; that is, to any member of  $\{A(AmB)\} - (\{AA\}g\{AB\})$ . Extensional disjunction, which is union, does not have this peculiarity because there is no extensional distinction between complete and incomplete sets. Obviously, normal linguistic usage conforms to extensional disjunction, not intensional.

Theorem 10 (6.10) gives the relationship between the two kinds of intensional disjunction, on the one hand, and possibility relations on the other.

That superintension is intensional implication will be much more clear in the next chapter, where *similarity truth* is defined (Def. 4.6) and superintension then requires the validity of argument forms such as *modus ponens*, *modus tollens*, and chain argument. Superintension may be thought of as **intensional analytic truth**, as opposed to *similarity truth*, or **synthetic truth**. The ancient definition of analyticity was that with it the predicate is contained in the subject; this was regarded as equivalent to the alternative definition that the denial of an analytic truth is, or leads to, a contradiction; but we can see now that the containment definition is intensional while the denial one is nominal. If S is called a subject, and a subintension, P, of it is called a predicate of it, so that  $S \circ P$ , then it is clear that superintension is a kind of containment, such that the subject contains the predicate. Putting this another way, S and P are abstract ideas, and any instance

of the first contains an instance of the second as a subintension. Because of this containment, S necessarily is a P; so, nominally, if it is denied that S is a P then S is both a P, because necessarily so, and not a P because of the denial. So nominal denial of an intensional analyticity produces a nominal contradiction. On the other hand, truth-functionally to deny a tautology such as PhP produces the contradiction PzUP, which is truth-functionally equivalent to U(PhP); but the tautology PhP will have no intensional meaning if P has exclusively nominal meaning, so that there is no superintension, no containment of predicate in subject, no intensional analyticity. So denial of an exclusively nominal analyticity, which latter is a tautology, produces a nominal contradiction; but there is no containment in exclusively nominal analyticity, unlike intensional analyticity. **Extensional analyticity**, it may be added, is subset: the subject set is contained in the predicate set; this follows from the implication theorem, (6.6).

That intensional equivalence is similarity, which is also symmetric implication, may seem implausible at first. Such implausibility is shown by the example of the concepts of equilateral triangle and equiangular triangle in Euclidean geometry. Each concept can be deduced from the other, so they are clearly equivalent, but they are equally clearly dissimilar, since *equilateral* does not mean *equiangular*. However, if L, A, and T stand for the intensions *equilateral*, *equiangular*, and *triangle*, then LnT and AnT stand for *equilateral triangle* and *equiangular triangle*; and LnT is such that from it emerges A, while from AnT emerges L. Thus each is more fully represented by the expression LnAnT, from which it follows that (LnT) o A and (AnT) o L. This gives us both the similarity and the symmetric implication, since superintension is intensional implication. In other words, the dissimilarity is nominal, not intensional.

\* \* \*

We are now in a position to consider truth tables for each of the three kinds of meaning, for each of the five connectives. We should note four preliminary points. First is that in a truth table such as the familiar Table 3.1 there are logical connectives between the columns, so each line of a truth table is a proposition. For example, in Table 3.1, the entries for a false material implication properly read as the proposition:  $(P \& UQ) \text{ s } U(PhQ)$ . So in the truth tables below these connectives are explicit. Second, in the extensional tables, T and F should be read as membership and non-membership (Def. 4.10), except when the column states an extensional fact, extensionally true or false, such as  $PiQ$  or  $P=Q$ . For example, the last line of extensional implication reads  $((xLP)r(xLQ)) \text{ s } (PkQ)$ . Third, in the intensional tables T and F should be read as similarity truth and dissimilarity falsity (Def. 4.6), which, speaking analogically, are the similarity or dissimilarity of a geographical map to its terrain. Fourth, all of the extensional truth tables can be verified with Venn diagrams, and all of the intensional truth tables can be verified with the kind of pattern diagram introduced earlier. Suitable diagrams are given in Fig. 3.3, in which  $\{AP\}$  is shown shaded with a positive slope and  $\{AQ\}$  with a negative slope.

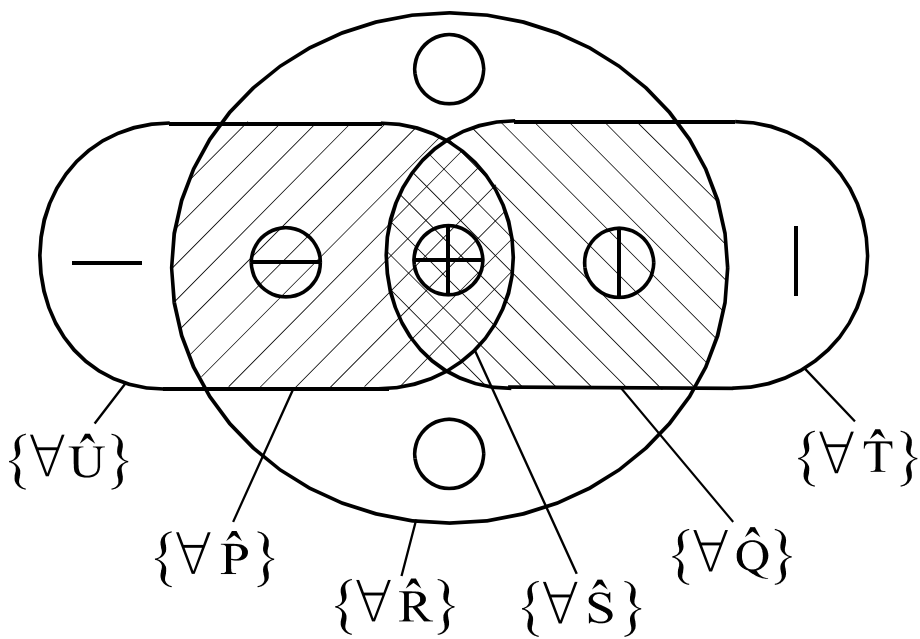
The truth tables for negation and conjunction are similar for all three kinds of meaning, which is the most likely explanation for the widespread but erroneous belief that all the nominal truth tables represent the natural meanings of the sentential connectives. In fact there are hidden dissimilarities between the truth tables for the three kinds of meaning for negation. First, extensional negation is set complementarity, and there is one instance where an extensional complement does not exist:  $U'=\phi$  does not exist in extensional set theory, since  $\phi$  belongs in nominal set theory only. Second, intensional negation is intensional complementarity, and this is very

rare. So the universality of nominal negation is not found in extensional and intensional negation.

In the case of disjunction all three truth tables are similar, with one exception: the intensional table disagrees with the other two in the case of false disjunction. This is because intensional disjunction has incomplete disjunction while extensional and nominal disjunction do not.

Extensionally,  $((xLP) \& (xLQ)) \supset (xL(P \vee Q))$ , as is shown in the Venn diagram of Fig. 3.3. But intensionally  $(P \vee Q) \supset R$  and  $(P \vee Q) \supset U$ , so if  $P \vee Q$  is false then  $P$  is false but  $P \vee Q$ , which is similar to  $U$  could be true, and similarly if  $Q \vee (P \vee Q)$ , which is similar to  $T$ , is false; so it is possible for both  $P$  and  $Q$  to be false, but for  $P \vee Q$  to be true — provided that the disjunction is incomplete. The diagram shows that it is possible for  $U$  and  $T$  to be false while  $R$  is true; and if  $U$  is false then  $P$  must be false, by *modus tollens* (4.3) (because  $P \supset U$ ), and similarly if  $T$  is false then so must  $Q$  be. For example, if  $R$  stands for *polygon*,  $T$  for *trilateral*, and  $U$  for *quadrilateral*, then  $T \supset R$ , or  $Q$ , stands for *triangle*, and  $U \supset R$ , or  $P$ , for *quadrangle*, and  $S$  does not exist; the intensional disjunction of these is  $((T \supset R) \vee (U \supset R)) \supset R$ , which is any polygon — and it is possible for something to be neither a triangle nor a quadrangle, yet still be a polygon.

The three truth tables for equivalence are similar with respect to their truth values, but otherwise the nominal truth table differs from the other two. This is because nominally there is universality: any two true propositions, or any two false propositions, are equivalent; and any false proposition is not equivalent to any true proposition. Intensionally



$$\hat{P} = \bigcirc \text{---}$$

$$\hat{R} = \bigcirc$$

$$\hat{U} = \text{---}$$

$$\hat{Q} = \bigcirc \text{---}$$

$$\hat{S} = \bigoplus$$

$$\hat{T} = \text{---}$$

Fig. 3.3.

and extensionally this is not so: propositions are intensionally equivalent only if they are similar, and they are extensionally equivalent only if they, as sets, are identical — in accordance with the equivalence theorem, (6.5).

In the case of implication the truth tables are not even similar with respect to their truth values: the intensional and extensional tables are similar to each other but dissimilar to the nominal table. The former are both ambiguous in the case of false antecedent and true consequent, and they both have true implication only when the antecedent and consequent are related by necessity. Therein lies the disagreement between nominal, or material, implication and our intuitive understanding of implication: we do not intuitively infer anything we please from a false proposition, nor allow anything we please to imply a true proposition.

The truth of **IqEqN** is easily verified with all of these truth tables.

\* \* \*

As illustrations of paradox having nominal meaning only, we next analyse Cantor's and Russell's paradoxes in terms of intensional, extensional, and nominal meanings.

Cantor's paradox arises from the definition of a power set: the power set of an  $n$ -membered set  $S$  is the set of all the subsets in  $S$ , including  $S$  itself, as an improper subset, and the null set; this power set is easily shown<sup>5</sup> to have  $2^n$  members. If now we define the set  $C$  as the set of all sets, and assume that  $C$  has  $c$  members, then  $C$  must have  $2^c$  members, since every member of the power set of  $C$  is a set; thus  $C$  must have more members than it has.

If we look at this intensionally, it is not immediately obvious whether the set of all intensional sets is itself an intensional set; but if it is not then the paradox cannot arise. So let us assume that the set of all

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<sup>5</sup> The number of ways in which  $k$  items may be selected from a set of  $n$  items, all different, is  $n!/(k!(n-k)!)$  so the number of  $k$ -membered subsets of an  $n$ -membered set is this number. So the total number of subsets is  $\sum n!/(k!(n-k)!)$  as  $k$  runs from 0 to  $n$ , inclusively. From the binomial theorem we know that  $(a+b)^n$  is  $\sum a^{n-k} b^k n!/(k!(n-k)!)$  as  $k$  goes from 0 to  $n$ , inclusively. So putting  $a=b=1$  gives the total number of subsets equal to  $2^n$ .

intensional sets is an intensional set, and call it  $C$ . Using orthodox symbolism for clarity,  $C = \{\{AX\}: X \text{ is an intension}\}$ . The subsets of  $C$ , other than those defined by  $X$ , which are themselves intensional sets take the form of  $\{A(AnB)\}$  or  $\{A(AmB)\}$ , where  $A|\{AX\}$  and  $B|\{AX\}$  are intensions, and provided that (i)  $(AnB)$  and  $(AmB)$  exist and (ii)  $\{AA\}g\{AB\}=\{A(AmB)\}$ . If these conditions are met,  $(AnB)$  and  $(AmB)$  are intensions and so  $(AnB)|\{AX\}$  and  $(AmB)|\{AX\}$ . Consequently the number of subsets of  $C$  is equal to the number of members of  $C$ , and Cantor's paradox has no intensional meaning.

Extensionally, the set of all extensional sets is itself not an intensional set because the term 'extensional set' does not have intensional meaning. So if it exists it is a contingent set, in which case it can only be defined by enumeration. The enumeration of it is itself a set of words, so the the enumeration must contain itself as a proper part; this is possible only with infinite sets, and an infinite ennumeration is impossible. So the set of all extensional sets does not exist. So Cantor's paradox has no extensional meaning.

It follows that Cantor's paradox has nominal meaning only.

As usually stated, Russell's paradox arises from the set,  $R$ , of all sets which are not self-membered, and its complement,  $R'$ , the set of all sets which are self-membered;  $R$  is itself either self-membered or not self-membered; if  $R$  is self-membered then  $R|R$ , and being a member of  $R$ ,  $R$  is not self-membered; and if  $R$  is not self-membered then  $R|R$ , hence  $R|R'$ , and being a member of  $R'$ ,  $R$  is self-membered. Stated symbolically:

$$(R|R)q(R|R')q(R|R) \text{ and } (R|R)q(R|R')q(R|R) \quad (3.11)$$

Intensionally, we note that self-membership is a monadic relation and so non-existent, and non-self-membership is the absence of a non-existent relation and so also non-existent: both are purely nominal relations: they have no intensional meaning. Hence  $R$  and  $R'$  do not have intensions and so cannot be intensional sets: their supposed intensions are nominal only.

Extensionally, since  $R$  and  $R'$  are not intensional sets, they must at best be contingent sets. But contingent sets cannot be either self-membered or non-self-membered, since these are purely nominal relations, so every contingent set is neither self-membered nor non-self-membered. So neither  $R$  nor  $R'$  can be enumerated, hence cannot be defined extensionally. Hence neither  $R$  nor  $R'$  has extensional meaning. Hence  $R$  and  $R'$  have nominal meaning only.

\* \* \*

A special kind of intensional meaning is:

**Def. 3.15**     **A relational meaning** is the relation that unifies an intensional set.

The relational meaning of an intensional set is distinct from each of: the set itself, the intension of the set, and the extension of the set. As a relation, it is an intensional meaning; but in many mathematically significant cases it is no more than a set relation (Def. [2.9](#)), having only the properties of simplicity and an adicity, and so not very significant. But in two other cases it is quite important: these are the cases of *compound relations* and *wholes*, which are dealt with in the next Chapter.

## 4. Mathematical Reasoning

Intensional sets are pluralities united by set relations, which latter have only the properties of simplicity and an adicity. Special cases of intensional sets are *compound relations* and *wholes*, which are united by relations having more properties than the simplicity and adicity possessed by a set relation.

**Def. 4.1**      A **compoundable relation** is any kind of relation that has one or more properties which are also possessed by the relation which unites a set of instances of that kind.

**Def. 4.2**      A **compound relation** is an extrinsic intensional set of compoundable relations, whose relational meaning (Def. 3.15) is of a kind similar to, or a superintension of, the kind of its members.

A compound relation thus differs from other intensional sets in that its unifying relation has a property set greater than that of a set relation: it has one or more properties similar to the properties of its terms. This unifying relation is the relational meaning of the set which is the compound relation; the extrinsic intension of this set is the intensional meaning of the compound relation, and the term set of this unifying relation is the extensional meaning of the compound relation. The unifying relation is not, of course, compound: like all relations, it is simple; the compoundedness of a compound relation belongs to its extensional meaning, or term set.

For example, we will later (Def. 5.21) define an *atomic length*, such that a set of contiguous atomic lengths has a compound length. *Atomic durations* may be compounded similarly. A *boundary* is a set of contiguous *dissimilarities*, such that what is bounded is dissimilar to its environment. A *process* is a sequence of causations, each of which

necessitates its effect, and such that the first transitively necessitates the last. More generally, all transitive relations (Def. 1.10) are compoundable relations, such that a compound of them is of the same kind. If  $T$  is a transitive relation such that  $aT_1b, bT_2c, \dots mT_nn$ , then  $aT_nn$  is a compound relation; and  $T_n$  is the relational meaning of this compound relation  $aT_nn$ , while  $\{T_1, T_2, \dots T_n\}$  is the extensional meaning, and the intensional meaning is the extrinsic intension based on the limits  $a$  and  $n$ . An *ordering* is also a compound relation, compounded out of many *ratios* (Def. 5.12), such as a well-ordered set of numbers, or a physical gradient such as a geographical gradient, a temperature gradient or a voltage.

Compound relations are like similarities and dissimilarities in that we have no criteria for which of them actually exist and which do not, and they multiply extravagantly. So we have to say, as with similarities and dissimilarities, that the existence of real compound relations is known by Occam's Razor: we multiply their existence only up to necessity.

Compoundable relations of dissimilar kinds may be compounded together: for example, an *atomic change* (Def. 5.25), is a compound of a dissimilarity, an atomic duration (Def. 5.22), and, in a deterministic context, a necessity or cause. Such a compound possesses each of the compoundable properties.

Compound relations are themselves compoundable. For example, a *degree of similarity* (Def. 5.26) is a compound relation compounded out of a compound similarity and a compound dissimilarity; and a *compound change* is a compound of compound dissimilarities, compound durations, and compound necessities.

A particularly important compound relation is an *arrangement*:

#### Def. 4.3

If an emergent relation  $R$  has a term set  $R$  then the **arrangement**,  $A$ , of  $R$  is the compound relation consisting of the set of all of those compoundable relations whose terms are members of  $R$  — all the compoundable upper extrinsic

properties of the terms of  $R$ , whose other terms are all terms of  $R$ .

We next consider the nature of a whole

**Def. 4.4** A **whole** is any emergent relation  $R$  which possesses at least one **novel property** — a property not possessed by any member of its term set,  $R$ , or any of its subordinate terms (Def. 5.15) — together with the term set  $R$  and the arrangement  $A$  of  $R$ . The members of  $R$  are called the **parts** of the whole;  $R$  — the relational meaning of the whole — is called the **top relation** of the whole; the subordinate terms of  $R$  are together called the **subordinate parts** of the whole if they are top relations of wholes; and the arrangement  $A$  is also called the **structure** of the whole, while structures of subordinate wholes are called **subordinate structures**.

A whole is a term set in being a plurality unified by a relation; it is complete. But a whole is more than this in that it has at least one novel property. Intensional sets which are unified by a set relation may be like a whole in having an arrangement — such as a compound relation — but do not have such a novel property. A whole is also more than a compound relation, in that a compound relation is unified by a relation that has more properties than a set relation, but its unifying relation does not have a novel property.

In short, an ordinary intensional set is unified by a set relation, a compound relation is an intensional set unified by a relation which has one or more of the properties of its terms, and a whole is an intensional set unified by a relation which has one or more novel properties. In this context *novel* means novel at that structure level, relative to all lower levels.

A simple machine is a whole consisting of a set of parts united by an emergent relation which has the property *in working order* — a property not possessed by any of its parts. A *melody* is a whole which

has notes as parts, and durations and musical intervals as members of its arrangement; and none of the notes are melodic.

Note that the relation *part of* is a genuine relation: the terms of  $R$  are not parts of  $R$  — which would make *part of* identical with *term of* and so a lower extrinsic property— but are parts of the whole: they are members of  $R$ .

We symbolise a whole by a boldface upper case letter, such as  $A$ ,  $B$ ,  $C$ , and the relation *part of* by  $<$ , and its inverse by  $>$ . Thus  $A < B$  means that  $A$  is a part of  $B$ , or  $B$  has  $A$  as a part.

**Def. 4.5**      An **element** of a whole is any one of: its emergent, or top, relation,  $R$ ; any member of the term set,  $R$ , of  $R$ ; any member of the arrangement,  $A$ , of  $R$ ; or any one of these of any of the subordinate parts of the whole.

As we have seen, two relations are similar if each member of the property set of each relation is similar to a member of the property set of the other relation. This concept may be extended to structures and wholes, as in the cases of congruence and isomorphism, matrix equality and algebraic structures. (See Defs. 5.26 and 5.33 for more details.) If two relations, or two structures, or two wholes are not similar then they are dissimilar.

**Def. 4.6**      If a relation, whole, or structure is a copy, representation, or reproduction of another, and they are similar, then their similarity is called the **intensional truth**, or **similarity truth**, of the copy, relative to the other, or original. If they are not similar then the copy is **intensionally false**, or **dissimilarity false**, relative to the original.

We symbolise this intensional truth by the symbol  $u$  and intensional falsity by  $U$ .

This definition of intensional truth is significant in two ways. One is that if a kind of an ideal relation,  $R_2$ , is similar to the kind,  $R_1$ , of

a real relation then the ideal kind is intensionally true:  $(R_2 \dagger R_1) \text{su} R_2$ . And what is true of kinds is also true of instances, so that if an ideal relation  $R_2$  is similar to a real relation  $R_1$  then the idea  $R_2$  is intensionally true:  $(R_1 \dagger R_2) \text{su} R_2$ . In the same way, since an intensional proposition is a whole composed of ideal relations — abstract ideas — then if an intensional proposition is similar to a portion of reality then the proposition is an intensionally true portion of applied mathematics. (A remark in passing: a necessary, but not sufficient, condition for the wholeness of a proposition is that the nominal expression of it must be a well-formed formula.) The second significance is that the principles of mathematical inference may be derived from this concept of truth, as follows.

**Def. 4.7**      The inference from **A** to **B** is **intensionally valid** if and only if  $(uA)q(uB)$  or  $(UA)q(UB)$ . Equally, the inference from **A** to **B** is an intensionally valid inference if and only if  $(uA)q(uB)$  or  $(UA)q(UB)$ ; the inference from **A** to **B** is an intensionally valid inference if and only if  $(uA)q(uB)$  or  $(UA)q(UB)$ ; and the inference from  $x \dagger A$  to  $x \dagger B$  is intensionally valid if and only if  $(uA)q(uB)$  or  $(UA)q(UB)$ .

Truth is a distributive property and falsity is a compositional property, as follows:

**Def. 4.8**      A relational property **D** is a **distributive property** if, given that a relation **R** possesses **D**, then so must each term of **R**.

**Def. 4.9**      A relational property **C** is a **compositional property** if, given that a term of a relation **R** possesses **C**, then so must **R**.

Definitions 4.8 and 4.9 clearly apply to intensional sets and thus to structures and wholes. We can then say that a property **D** is distributive to a set if, given that the unifying relation of the set has **D**, then each member of the set must also possess **D**; and if a member of a

set has a property **C** then **C** is compositional if the unifying relation of the set must also possess **C**. And similarly for wholes and their parts. We can in fact go further and say this is true of arrangements as well, so that if a whole **W** has **D** then so does every element of **W**, while if an element of **W** has **C** then so does **W**.

Let **D** be a distributive property. First, if **C** is the absence of **D** then **C** must be a compositional property; second, if **D** and **C** are mutually exclusive then the presence of **C** requires the absence of **D**, so **C** is compositional; third, if **C** and **D** are complementary then they are mutually exclusive; hence the absence or complement, **C**, of **D** is compositional. Conversely, by similar reasoning, if **C** is compositional and the absence or complement of **C** is **D**, then **D** must be distributive.

The absences of **D** and of **C** are only nominal properties, but as such they each still deny the other. Thus complementary distributive and compositional properties are skew-separable (Def. 1.1): a compositional property can be present in the term without it being present in the relation because the term may exist without the relation (although if the relation exists then it must have the compositional property); but the complement of this compositional property is a distributive property which cannot be present in the relation without it being present in the term.

Two clear examples of distributive and compositional properties are existence itself, and consistency: if a relation exists then so does each of its terms, if a set exists then so does each of its members, and if a whole exists then so does each of its elements; and if one term or member or element does not exist then neither do the relation or the set or the whole. Similarly, if a relation, set, or whole is consistent then so is each term, member, or element of it, while if one (nominal) term, member, or element is inconsistent then so is the (nominal) relation, set, or whole.

Similarity and dissimilarity, which are intensional complements of each other, are respectively distributive and compositional (upper extrinsic) properties. If a whole **A** is similar to a whole **B** then each element of **A** is similar to an element of **B**, while if one or more

elements of **A** are dissimilar to their corresponding elements (Def. 5.33) of **B** then the whole of **A** is dissimilar to **B**. So the similarity of **A** to **B** is a distributive property of **A** (and of **B**, since similarity is symmetric), and the dissimilarity of **A** to **B** is a compositional property.

It follows that intensional truth and falsity, each of which is the intensional complement of the other, are respectively a distributive property and a compositional property. Consequently intensional truth and falsity apply, as distributive and compositional properties, to superintension of property sets, to superset of extensions, and to the relation of *part of* between whole and part, including elements as parts. In each case we get the basis of accepted valid rules of inference.

In the case of superintension, such as S oP, we get:

$$(S \text{ o}P)s((uS \text{ q}uP) \text{ or } (UP \text{ q}US)). \quad (4.1)$$

It follows that if S oP and uS, then necessarily uP:

$$((S \text{ o}P)\&uS) \text{ q}uP; \quad (4.2)$$

and if S oP and UP then necessarily US:

$$(S \text{ o}P)\&UP) \text{ q}US; \quad (4.3)$$

thus we have the validity of intensional *modus ponens*, or affirmation of the antecedent, and of *modus tollens*, or denial of the consequent.

In the case of superset of extensions, such as ShP, we get:

$$(ShP)s((uS \text{ q}uP) \text{ or } (UP \text{ q}US)). \quad (4.4)$$

Hence

$$((ShP)\&uS) \text{ q}uP \quad (4.5)$$

and

$$((ShP)\&UP) \text{ q}US. \quad (4.6)$$

In the case of *part of*, such as  $\mathbf{S} > \mathbf{P}$ , we get:

$$(\mathbf{S} > \mathbf{P})s((u\mathbf{S}q u\mathbf{P}) \text{ or } (u\mathbf{P}q u\mathbf{S})). \quad (4.7)$$

So

$$((\mathbf{S} > \mathbf{P}) \& u\mathbf{S})qu\mathbf{P} \quad (4.8)$$

and

$$((\mathbf{S} > \mathbf{P}) \& u\mathbf{P})qu\mathbf{S}. \quad (4.9)$$

The three kinds of symbolism are needed in depicting argument forms precisely. Property sets or single properties, such as  $\mathbf{S}$  and  $\mathbf{P}$ , and the set relations of intensional sets such as  $\mathbf{S}$  and  $\mathbf{P}$ , are the intensional meanings of concepts, as we shall see later, while wholes such as  $\mathbf{S}$  and  $\mathbf{P}$  are the intensional meanings of either concepts or propositions — propositions being wholes consisting of structures of concepts. For example,  $\mathbf{S}$  might be a concept such as *triangle* — a triangle being a whole (since its top relation has a property, area, not possessed by any of its parts) — or a proposition such as Pythagoras' theorem. Also, the intensional connectives are not always clear in the case of wholes. If a whole  $\mathbf{R}$  has the top relation  $\mathbf{R}$  which has the property set  $\mathbf{R}$ , term set  $\mathbf{R}$ , and arrangement  $\mathbf{A}$ , and if it is a fact that  $\mathbf{R} \circ \mathbf{P}$ , then the proposition that  $\mathbf{R}$  is a  $\mathbf{P}$  is true; but  $\mathbf{R} \circ \mathbf{P}$  is not defined, so that the statement  $\mathbf{R} \circ \mathbf{P}$  is to be preferred. However, rather than duplicate all our expressions of argument forms, we will continue to use the symbolism of property sets, with the understanding that the symbolism for sets and wholes may be substituted whenever the connectives are defined.

The intensional validity of hypothetical syllogism, or chain argument,  $(u(\mathbf{A} \circ \mathbf{B}) \& u(\mathbf{B} \circ \mathbf{C}))qu(\mathbf{A} \circ \mathbf{C})$ , and of simplification,

$u(AnB)quA$  and  $u(AnB)quB$ , is easily shown, since *superintension* is transitive; and, in the case of simplification, it is true that  $(AnB) oA$ .

Another intensionally valid argument form is that of the principle of substitution of equivalents: if  $PtQ$  then  $P$  may be substituted for  $Q$  in a proposition, and *vice versa*. This is due to the fact that

$$(PtQ)s((uPsuQ) \text{ or } (UpsUQ)) \quad (4.10)$$

(see Table 3.6; (4.10) differs from (4.1) in that the right hand side of (4.10) has symmetric necessities in place of the asymmetric ones of (4.1)). Expression (4.10) derives from the fact that intensional truth is similarity, which is transitive:  $uP$  means that  $PtR$ , where  $R$  is that kind of real relation that  $P$  resembles, so  $PtQ$  means that  $QtR$ , hence  $uQ$ , and *vice versa*; and similarly for  $UpsUQ$ .

Inference of non-existence by *reductio ad absurdum* is a case of *modus tollens*, based on the fact that a contradiction is necessarily false. However, although proofs of non-existence by *reductio* are valid, proofs of existence by *reductio* are not, since the existence so proved may be only nominal existence. This point is a justification of the intuitionists' claim that existence proofs require constructions — if the existence intended is intensional existence. A construction is, of course, a whole that is ideal.

The traditional laws of thought all follow from the concepts of intensional truth and falsity applied to intensional meanings. These are the rules of identity, excluded middle, and non-contradiction:  $uPquP$ ,  $Pv(uP|UP)$ , and  $U(uP\&UP)$ , respectively.

Note that in the rule of identity the necessity is not monadic, since it relates two instances of the truth of  $P$ , rather than one instance to itself. Idempotence is possible with intensions: the equivalence theorem (6.5),  $(AtB) s (\{AA\}=\{AB\})$ , shows that

idempotence may have intensional meaning when applied to intensions, but not when applied to intensional sets. This theorem requires that there cannot be two similar intensional sets, for if  $\{AA\} \neq \{AB\}$  then  $A \neq B$ , in which case  $\{AA\} = \{AB\}$ ; this means that  $\{AA\}$  and  $\{AB\}$  are identical, they are one. So to say that  $A \cap A = A$  requires that union and intersection may be monadic — and there are no monadic relations. Such idempotence is acceptable in nominal set theory, because there nominal monadic relations are allowed. This idempotence applies to reflexive relations (Def. 1.11), provided that the relation concerned has more than one instance. If  $n$  is an intensional number (see Chapter 5) then  $n = n$  means that one instance of  $n$  is equal to another; but the extensional number  $n$ , which is the set of all sets that are equiadic with an  $n$ -membered set, is not equal to itself; and if  $n$  is an exclusively nominal number then  $n = n$  is acceptable, by analogy to the intensional case.

Another point is the intuitionists' claim that their criterion of constructability for mathematical existence requires the denial of the principle of excluded middle: some propositions are neither true nor false. In the present context such a proposition is one containing one or more entities that cannot be constructed; which is to say that it is an exclusively nominal proposition. It is obvious that every intensional proposition is either intensionally true or intensionally false, and every nominal proposition is either nominally true (Def. 4.13) or nominally false; but, equally clearly, not every nominal proposition is either intensionally true or intensionally false, since exclusively nominal propositions are neither. This last is the basis of the intuitionists' denial of excluded middle. For example, "Every monadic relation is self-similar" is neither intensionally true nor intensionally false, since it has no intensional meaning, but it is nominally true.

We have, of course, been presupposing all of these argument forms, so that we cannot escape circularity here; but we are not

defining or asserting the argument forms, we are explaining them, with the aim of distinguishing them from inferior, nominal, versions; so the circularity is not vicious.

The significance of this validity for mathematics is that since theorems are emergent out of axiom sets, and emergence is extrinsic necessary existence, it follows that if an axiom set is intensionally true then so are every one of its theorems; and since the truth of the axioms necessitates the truth of the theorems, the theorems are validly implied by the axioms, hence deducible from the axioms.

There are four standard argument forms that are intensionally valid only in limited circumstances.

First, addition,  $(uP \& uQ) \supset (P \vee Q)$  is valid only if the coupling is possible.

Second, disjunctive syllogism,  $(u(P \vee Q) \& uP) \supset Q$  or  $(u(P \vee Q) \& uQ) \supset P$ , is valid only if the disjunction is complete; so unless the (intensional) completeness can be established first, this argument form is intensionally invalid. Using our earlier examples of complete and incomplete disjunction, we can say that if something is a natural number and it is not an odd number, then it is an even number:  $(u((NnO)m(NnE)) \& U(NnO)) \supset (NnE)$ ; but if something is a polygon and it is not a triangle, it does not follow that it is a quadrilateral:  $(u((TnP)m(QnP)) \& U(TnP)) \not\supset (QnP)$ .

Third, disjunctive addition,  $uP \supset (P \vee Q)$ , is intensionally valid only if the commonality  $PmQ$  exists.

Fourth, contraposition,  $u(A \supset B) \supset u(B' \supset A')$ , is intensionally valid only if both of the intensional complements exist.

\* \* \*

We may compare all this with extensional and nominal truth and validity.

There are two ways to discover the nature of extensional truth and falsity, which is useful because they are peculiar. The first way is to derive it from intensional truth and falsity, through the implication and equivalence theorems.

The implication theorem (6.6) is  $(A \circ B) \text{ s } (\{AA\} \text{ k } \{AB\})$ . Since superintension is the basis of intensional validity, subset must be the basis of extensional validity. But for all  $x$ ,  $(SiP) \text{ s } (((x \text{ l } S) \text{ q } (x \text{ l } P)) \text{ or } ((x \text{ l } P) \text{ q } (x \text{ l } S)))$ . This is analogous to (4.1):  $(S \text{ o } P) \text{ s } ((U \text{ S } \text{ q } U \text{ P}) \text{ or } (U \text{ P } \text{ q } U \text{ S}))$ , from which we can infer that extensional truth is membership and extensional falsity is non-membership. The equivalence theorem (6.5) shows the same thing:  $(A \text{ t } B) \text{ s } (\{AA\} = \{AB\})$  makes set identity the extensional meaning of equivalence; but for all  $x$ ,  $(S = P) \text{ s } (((x \text{ l } S) \text{ s } (x \text{ l } P)) \text{ or } ((x \text{ l } P) \text{ s } (x \text{ l } S)))$ , which is analogous to (4.10),  $(P \text{ t } Q) \text{ s } ((U \text{ P } \text{ s } U \text{ Q}) \text{ or } (U \text{ P } \text{ s } U \text{ Q}))$ .

The second way to find the meaning of extensional truth and falsity is from the supposition that a meaningless sentence is false. A set is extensionally meaningless if it is null, so extensional falsity is absence of members and extensional truth is thereby possession of at least one member. This is clear when expressed in the symbols of quantificational logic: that  $S$  is null is stated by  $(x)(x \text{ l } S)$ ; and  $(x)(x \text{ l } S) \text{ s } U(E \text{ x})(x \text{ l } S)$ : that is,  $S$  is extensionally false means that  $S$  is not extensionally true. Thus if  $S$  has at least one member,  $(E \text{ x})(x \text{ l } S)$ , then  $S$  is extensionally true. So we have:

**Def. 4.10**      **Extensional truth** is set membership, and **extensional falsity** is non-membership.

For example, if  $N$  is the set of natural numbers then if  $n \text{ l } N$  this means that it is true that  $n$  is a natural number, while  $p \text{ l } N$  means that it is false that  $p$  is a natural number.

**Def. 4.11** An **extensional inference** from  $A$  to  $B$  is  $A \rightarrow B$  and is **extensionally valid** if and only if either (i) membership in  $A$  is universally membership in  $B$ : always if  $x \in A$  then  $x \in B$ ; or (ii) non-membership in  $B$  is universally non-membership in  $A$ : always if  $x \notin B$  then  $x \notin A$ .

**Def. 4.12** **Extensional equivalence** is set identity, more commonly known as set equality, and is **extensionally valid** if and only if either (i) membership in  $A$  is universally membership in  $B$ , and *vice versa*: always if  $x \in A$  then  $x \in B$ , and if  $x \in B$  then  $x \in A$ , or (ii) non-membership in  $B$  is universally non-membership in  $A$ , and *vice versa*: always if  $x \notin B$  then  $x \notin A$ , and if  $x \notin A$  then  $x \notin B$ .

Thus extensional validity is based on extensional necessity, not on intensional necessity.

The intensionally valid argument forms are all extensionally valid, and as well addition, disjunctive syllogism, disjunctive addition, and contraposition are extensionally valid, as is easily shown by Venn diagrams. Cast in extensional symbols, these argument forms are:

<i>Modus ponens</i> :	$((S \rightarrow P) \& (x \in S)) \rightarrow (x \in P)$
<i>Modus tollens</i> :	$((S \rightarrow P) \& (x \notin P)) \rightarrow (x \notin S)$
<i>Hyp. syllogism</i> :	$((S \rightarrow P) \& (P \rightarrow R)) \rightarrow (S \rightarrow R)$
<i>Simplification</i> :	$(x \in S \& P) \rightarrow (x \in S)$
<i>Substitution</i> :	$(S = P) \rightarrow ((x \in S) \rightarrow (x \in P)) \text{ or } ((x \notin P) \rightarrow (x \notin S))$
<i>Addition</i> :	$(x \in P) \& (x \in S) \rightarrow (x \in P \vee S)$
<i>Disj. syllogism</i> :	$((x \in S \vee P) \& (x \notin P)) \rightarrow (x \in S)$
<i>Disj. addition</i> :	$(x \in S) \rightarrow (x \in S \vee P)$
<i>Contraposition</i> :	$((x \in S) \rightarrow (x \in P)) \rightarrow ((x \notin P) \rightarrow (x \notin S))$

**Def. 4.13**      **Nominal truth** is correct statement of meaning —  
intensional, extensional, or nominal, ideal or real — and  
**nominal falsity** is incorrect statement, as in error or deceit.

*Correct* and *incorrect* here refer to the established use of language. Language that has no established use has no nominal meaning. So “All square circles are circles” has nominal meaning, by analogy to “All right triangles are triangles”, but “All squircles are cirquare” has no nominal meaning, hence no meaning at all. We note, however, that nominal meaning is easily established by stipulative definition: if we define a squircle as a square circle and a cirquare as a circular square, then “All squircles are cirquare” both has nominal meaning and is nominally true.

**Def. 4.14**      A nominal inference of one statement,  $Q$ , from another,  $P$ , is  
**nominally valid** if and only if the truth function  $PhQ$  is  
tautologous, or always nominally true.

With all three kinds of inference a true premise and false conclusion are a sufficient condition for invalidity. If  $uS$  and  $UP$  then it is impossible that  $S \text{ OP: } P$  cannot be intensionally validly inferred from  $S$ . If  $xIS$  and  $xLT$  then it is impossible that  $SiT: xIT$  cannot be extensionally validly inferred from  $xIS$ . And if  $PzUQ$  then it is impossible that  $PhQ$  is true:  $Q$  cannot be nominally validly inferred from  $P$ . And intensional validity is a sufficient condition for extensional validity, which is a sufficient condition for nominal validity; but nominal validity is only a necessary condition for extensional validity, which is only a necessary for intensional validity: **IqEqN**. Thus all inferences from purely nominal statements are nominally valid, but have no extensional or intensional validity; all inferences from membership in a random subset,  $S$ , of a random set  $T$ , to membership in  $T$  are extensionally and nominally valid, but

have no intensional validity; and all inferences from an intension S to one of its subintensions, P, are intensionally, extensionally, and nominally valid.

As an illustration of the value of separating intensional validity from nominal validity, we consider the well known nominal proof that from a contradiction we may validly deduce anything we please:

1. PzUP	Premise	
2. P	1, Simplification	
3. UP	1, Simplification	
4. PyQ	2, Disjunctive addition	
5. Q	4, 3 Disjunctive syllogism.	(4.11)

Intensionally the argument is invalid both because, first, the disjunctive syllogism is only valid if both the commonality of P and Q exists and the disjunction can be shown to be complete — which is impossible if Q is any proposition whatever; and, second and more significantly, the argument is intensionally invalid because the premise has no intensional meaning. Nominally, of course, the argument is valid.

The inadequacy of the truth-functional basis of logic has long been known; it is clearly illustrated by (4.11) and by the theorems in truth-functional logic that a false proposition implies any proposition, a true proposition is implied by any proposition, and any two true propositions are equivalent, as are any two false propositions — theorems that have nominal meaning only. And also by the fact that if PhQ then by addition there is an infinity of premises from which Q may be deduced, and by disjunctive addition there is an infinity of conclusions to be drawn from P. These inadequacies are not inadequacies of Boolean algebra, they are the inadequacy of applying this algebra to a truth-functional logic.

## 5. Foundations

Having examined the intensional, extensional, and nominal foundations of set theory and logic, we now do the same for the foundations of elementary arithmetic, geometry, applied mathematics, and fuzzy set theory and fuzzy logic.

We have seen that every relation, without exception, necessarily has a term set, and has the properties of simplicity and an adicity: relations without these are impossible, merely nominal. And we also noted, metalinguistically, that the adicity of a relation is the number of terms that it has, the number of members in its term set. We now state this formally:

**Def. 5.1**      An **intensional natural number** is an adicity, with the exception of the natural number one.

We will symbolise the natural number of a relation  $R$  by the same letter, lower case, italic:  $r$ . That is,  $r$  is the adicity of  $R$ . (Because  $r$  is a property we should symbolise it as a subintension of  $R$ ; but we will stay with the more conventional usage.) Obviously, the natural number of the term set,  $R$ , of a relation  $R$  is the adicity,  $r$ , of  $R$ . Because there are no monadic relations there are no one-membered term sets. So the intensional natural number one is not an adicity. (There are one-membered contingent sets, but these are of no use in an intensional arithmetic.)

Every set relation is an instance of its own adicity, and every other relation is a representative instance of its own adicity.

**Def. 5.2**      The **intensional natural number one** is the commonality of every intrinsic property set, the commonality of every relation.

The number one is thus the property set consisting of adicity in general — as opposed to a particular adicity — and simplicity.

Adicity in general is number, and simplicity is unity, so this property set has only two members, and the two together constitute the number one. Since this is a subintension of every property set, every relation is a unity; so any instance of any relation may be a representative instance of the intensional natural number one — as is shown by us calling it one relation. The number one should not be confused with set relations, each one of which has, besides simplicity, a particular adicity.

Thus intensionally there is a fundamental difference between singularity and plurality: it is both the difference between a relation, which is a unity, and its term set, which is a plurality; and the difference between identity and similarity. This is echoed in the grammar of ordinary language, and was claimed by the ancient Greeks; and it is a natural justification for the standard claim that the number one is neither prime nor composite, in order to preserve the fundamental theorem of arithmetic<sup>6</sup>.

The key features of an intensional arithmetic now follow in an obvious and natural way.

**Def. 5.3** Two relations are **equiadic** if each is a representative instance (Def. 2.30) of the other's adicity.

**Def. 5.4** Two intensional natural numbers  $m$  and  $n$  are **equal**, symbolised  $m=n$ , if they have representative instances  $M$  and  $N$  which are equiadic.

**Def. 5.5** An intensional natural number  $m$  is **greater than** another,  $n$ , symbolised  $m>n$ , if there exist representative instances  $M$  and  $N$  whose term sets are such that  $N \cap M$ . The inverse of *greater than* is **less than**, symbolised by  $<$ .

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<sup>6</sup> The fundamental theorem of arithmetic is the theorem that any positive integer (other than one) can be written as a product of primes in one, and only one, way.

It is not necessary for the relation of subset to exist between two term sets  $R$  and  $S$  in order to have a relation of *greater than*: if  $R$  and  $S$  are such that  $R \leq S$  and  $S \leq R$ ,  $R$  is equiadic with  $M$  and  $S$  is equiadic with  $N$ , and  $M \leq N$ , then  $r > s$ .

**Def. 5.6** If two relations  $M$  and  $N$  have disjoint term sets  $M$  and  $N$ , and natural numbers  $m$  and  $n$ , and there exists a relation  $R$  having the term set  $M \cup N$  and adicity  $r$ , then the **sum** of  $m$  and  $n$ , symbolised  $m+n$ , is the number  $r=m+n$ . If there exists a relation  $S$  of adicity  $s$  whose term set  $S$  consists of  $M$  and a single other relation, then the sum  $m+1$  is the number  $s=m+1$ .

**Def. 5.7** The **addition** of two numbers  $m$  and  $n$  is the intensional binary operation, or function, having the set of the two of them as its argument and their sum as its value.

**Def. 5.8** If an intensional natural number  $m$  is greater than another,  $n$ , the **difference** between them, symbolised  $m-n$ , is the adicity of the difference of their term sets,  $M-N$ .

The same proviso as that of Def. 5.5 applies to this definition.

**Def. 5.9** The **subtraction** of a number  $n$  from another number  $m$ , where  $m > n$ , is the intensional binary operation, or function, having the ordered pair of them,  $(m, n)$ , as its argument and their difference as its value.

**Def. 5.10** The **multiplication** of two numbers,  $m$  and  $n$ , symbolised  $m \times n$  or  $mn$ , is the repeated addition of  $n$  instances of  $m$ :  $m \times n = m_1 + m_2 + \dots + m_n$ .

**Def. 5.11** The **division** of two numbers,  $m$  and  $n$ , such that  $m > n$ , symbolised  $m/n$ , is the repeated subtraction of instances of  $n$

from  $m$  until no further subtraction is possible; the number of subtractions,  $p$ , is the quotient of the division, such that  $m/n=p$ . If after the  $p$  subtractions there remains a number  $q<n$ , then  $q$  is the **remainder**.

**Def. 5.12** We postulate a bipossibility relation called a **comparison of numbers**, whose antecedent is always an ordered pair of intensional natural numbers, and whose consequents are *equal* and *unequal*; if unequal, the consequent is either *larger than* or *smaller than*, depending on the ordering of the antecedent. The comparison of two numbers,  $m$  and  $n$ , is then the **ratio** of those numbers, symbolised  $m:n$ , or, more conventionally,  $m/n$ .

Ratios are compoundable relations, their compounds being *orderings*.

**Def. 5.13** A ratio  $a:b$  is **equal** to another ratio  $c:d$  if and only if  $ad=bc$ .

**Def. 5.14** A ratio  $a:b$  is **larger than** another ratio  $c:d$  if and only if  $ad>bc$ , and  $a:b$  is **smaller than**  $c:d$  if and only if  $ad<bc$ .

A relation defines more than just one number with its adicity, since it defines the sum of the adicities of its terms, and of their terms, on down.

**Def. 5.15** The terms of a relation  $R$ , which are themselves relations, are said to be one **level** lower than the level of  $R$ . The terms of the terms of  $R$ , the terms of their terms, and so on down to the lowest level, are together called the **subordinate terms** of  $R$ .

**Def. 5.16** If a relation  $R$  has adicity  $r$ , then the **first level subordinate adicity** of  $R$  is the sum of the adicities of each of the  $r$  terms

of R: the total number of the terms of the terms of R. The **second level subordinate adicity** of R is the total number of the terms of the terms of the terms of R. The **prime level subordinate adicity** of R is the lowest level subordinate adicity of R (Def. 5.19).

If a whole **W** has a top relation T then the sum of all the subordinate adicities of T, plus the adicity of T, plus 1, is the number of relations in **W**, other than relations in arrangements (Def. 4.3).

These definitions in intensional arithmetic lead to several observations concerning intensional natural numbers.

First, the similarity set of a natural number is the set of every relation that is equiadic to it. It follows that there is only a nominal difference between a particular number and various instances of it: the number is the intension of the similarity set and the totality of instances of it are the similarity set, but each instance, with **t**, serves as an intension of the set. Thus in an expression such as  $2+2=2\times 2=2^2$ , each of the six instances of 2 is the number 2, since, with **t**, it serves as an intension of the similarity set of all dyadic relations.

Second, there is no intensional natural number zero: the word zero has only nominal meaning. We could define it as the adicity of any “nonadic” relation, but none such exist — they are at best purely nominal relations. Or we could define it as the extensional number (defined shortly) of the null set, but there are no null sets in extensional set theory: the null set, which is the intersection of non-intersecting sets or the extension that is not an extension, is self-contradictory and so has only nominal meaning. So, having neither intensional nor extensional meaning, zero has only nominal meaning. This shows that nominal meaning can have practical utility in mathematics, as with the value of zero in positional numeration — even though it threatens nonsense, as with division by zero, and has peculiarities, such as  $0!=n^0=1$ .

Third, there are no infinite intensional numbers. The standard definition of an infinite number is that it is the number  $\aleph$  of a set which possesses at least one proper subset having an equal number  $\aleph$ . If we suppose that  $M$  and  $N$  are two such intensional sets, such that  $N \ni M$ , then the fact that  $n=m$  requires that  $N \ni M$ . This contradiction requires that  $\aleph$  has only nominal meaning<sup>7</sup>. Also, Zeno's paradoxes, which were designed to show the impossibility of motion, given infinite divisibility, in fact show by *reductio* that because motion does in fact exist, infinite divisibility is impossible.

Fourth, in order to define the operation of addition of intensional natural numbers an assumption is required:

### **The axiom of addition:**

If there exist two relations  $R$  and  $S$ , having disjoint term sets  $R$  and  $S$  and adicities  $r$  and  $s$ , then, with certain exceptions, there exists an intensional set equiadic with  $R \cup S$ , so that the number  $r+s$  exists.

The exceptions to this axiom arise because, if there are no intensional infinities then there must be a greatest finite intensional natural number: the number of relations in the largest whole<sup>8</sup>, as will become clear in Chapter 8. Call it  $g$ . Then if  $m < g$  and  $n < g$  are intensional natural numbers then  $n+m$  is an intensional natural number only if  $(n+m) \leq g$ . Thus there is no closure on intensional

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<sup>7</sup>Bertrand Russell, in discussing the one-to-one correspondence between the set of natural numbers and the set of even numbers, wrote: "Leibniz, who noticed this, thought it a contradiction ... Georg Cantor, on the contrary, boldly denied that it is a contradiction. He was right; it is only an oddity." (*A History of Western Philosophy*, Allen and Unwin, London, 1946, p.858.) As much as I admire Russell, I have to agree with Leibniz.

<sup>8</sup>The total number of relations is this number plus all the relations in all of the arrangements of their terms; this is an extensional number.

addition. If  $g$  is large beyond our comprehension<sup>9</sup>, this non-closure is of no practical significance.

Fifth, it might be thought that because  $m+n$  is defined by  $MgN$ , then, by the disjunction theorem (6.9),  $\{A(AmB)\}j\{AA\}g\{AB\}$ , it follows that  $(m+n)t(mmn)$  — recalling that  $m$  and  $n$  are properties of the relations  $M$  and  $N$ . Since the commonality of  $m$  and  $n$  would seem to be adicity *per se*, this would make nonsense of the sum of  $m$  and  $n$ . However, we observe that the extensional disjunction  $MgN$  is incomplete, hence  $(m+n)o(mmn)$ : the legitimate sum of two intensional natural numbers is a superintension of adicity — obviously, since every intensional number is a superintension of adicity.

Sixth, we may note that Godel's theorems, on the incompleteness and consistency of any system large enough to contain number theory, are not proved in a purely intensional arithmetic because it is finite, so it cannot be large enough to map the requisite formulas into itself. In fact, since inconsistent mathematics has nominal meaning only, it follows that purely intensional arithmetic *must* be consistent. Thus Hilbert's program of proving the consistency of arithmetic is restored in intensional arithmetic: the program requires only the separation of intensional mathematics from nominal mathematics — not, of course, an easy task. Also, as we shall see in Chapter 8, intensional mathematics *must* be complete as well as consistent.

Seventh, since the truth of intensional mathematics is relative to reality, intensional mathematics is applied mathematics.

Finally, on intensional natural numbers, we may paraphrase Kronecker's famous dictum and say that God made the intensional

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<sup>9</sup>As an example of an incomprehensibly large finite number, consider the function  $+^n$ , defined recursively by:  $(a +^n b)$  has the value of  $b$  instances of  $a$  related by  $+^{n-1}$ , where  $a$  and  $b$  are intensional numbers, and  $(a +^n b)$  has the values  $a+b$ ,  $a \times b$ , and  $a^b$  for  $n=1, 2$ , and  $3$ . Then if a number such as  $m=(100 +^{100} 100)$  is not incomprehensibly large, then  $(m +^m m)$  is.

numbers and that all the extensional numbers and all the nominal numbers (see below) are the work of man. One could define intensional rational numbers as ratios, but from an intensional point of view a ratio is not a number. One could also define negative numbers by means of the sense of vectors, and imaginary numbers by defining multiplication by  $i$  as the anticlockwise rotation of a vector through a right angle, but intensionally these are not adicities. The key point here is that to define something mathematically does not ensure that it has intensional meaning, and to specify principles of closure, on addition, subtraction, division, the taking of roots, etc., does not ensure that these principles are intensionally true.

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We turn next to extensional and nominal arithmetic.

It should be clear that if two relations are equiadic then their term sets are in one-to-one correspondence, in the sense that two contingent sets  $A$  and  $B$  are in one-to-one correspondence if there is a contingent function from every member of  $A$  to a member of  $B$ , and *vice versa*. From this we may extend the definition of an intensional number to incomplete sets: the intensional natural number of an incomplete set is the number of any complete set with which it is in one-to-one correspondence.

**Def. 5.17**      The **extensional natural number** of an extension is the set of all extensions with which it is in one-to-one correspondence.

Extensional arithmetic now follows in the usual way.

**Def. 5.18**      A **nominal number** is any intensional or extensional natural number or any number defined nominally out of these natural numbers.

Nominal arithmetic includes numbers that have exclusively nominal meaning. One such is the number zero which, as already explained, has exclusively nominal meaning. Another class of numbers that have exclusively nominal meaning is the infinite numbers. They do not have intensional meaning, as already explained, so unless it can be shown that there are contingent sets which have an infinite number of members, there is no alternative but to say that infinities have exclusively nominal meaning. Contingent sets may be constructed by union of intensional sets, or by forming power sets, but neither the union nor the power sets of a finite number of finite intensional sets can yield an infinite set. Alternatively, contingent sets may be defined by enumeration, but an infinite enumeration is impossible. Hence all infinite numbers are exclusively nominal.

Thus purely nominal numbers are fictions: often very useful fictions, to be sure, but a utility in which paradox lurks.

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In order to consider intensional geometry we need the concept of prime relation.

**Def. 5.19**      Relations at the lowest level are called **prime relations**, or **separators**.

Prime<sup>10</sup> relations are compoundable relations; they have properties called **unit magnitude**, or **unit measure**, which sum. Compound relations formed from them have magnitudes of the same kind, equal to their adicity.

The terms of prime relations are not lower level relations, by definition, so they must be relations of the same level. They may be

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<sup>10</sup> The word *prime* used here originates, not from prime numbers, but from Aristotle's concept of prime matter.

thought of as separators, in the sense that separators separate separators, like an unending line of hyphens: each hyphen separates two hyphens, and is separated by each of these from another two. Since each separator is indifferently both relation and term, they must all be at the same level.

We adopt the convention that this lowest level is level-1; thus emergent relations that have level-1 relations as their terms are level-2 relations, and so on up. Level-1 is also called the **prime level**.

It is characteristic of prime relations that with a first level compound of them the extension and the arrangement of this compound relation are identical.

The alternative to having prime relations is an infinite regress of lower and lower levels. One reason for denying such an infinite regress of levels is that we have already denied infinities. A more specific reason is that prime relations need to have properties which sum: properties such as length and angle, out of which a geometry should emerge. If the prime relations that possess the magnitude called length were geometric points, which is what an infinite regress would require, then they would not have a magnitude that sums. Another reason that points could not be separators is that no point in the traditional continuum has a point next to it because between any two points there is an infinity of points, so that a point cannot immediately separate two other points; thus geometric points cannot be relations, so cannot have intensional meaning. Infinite divisibility has nominal meaning only.

Prime relations provide, in principle, a foundation for both geometry and applied mathematics. We begin with geometry. We assume two kinds of prime relations, each possessing magnitude:

**Def. 5.20**      **A linear separator** has unit length and is symmetric.

**Def. 5.21**      **An angle separator** is a right angle and is symmetric.

A linear separator may also be thought of as an atomic length — using *atomic* in its original Greek sense of an indivisible. If four linear separators and four angle separators are combined so that each of the linear separators separates two of the angle separators, and each of the angle separators separates two of the linear separators, then there emerges a new relation: a level-2 octadic atomic square, having the emergent property of **atomic area**. Atomic cubes and volumes emerge similarly. Atomic areas are compound relations: they are emergent separators which separate other atomic areas, as atomic volumes also are separators separating other atomic volumes; and areas and volumes are compoundable relations that sum, so that other lengths, areas, and volumes are sums of integral numbers of atomic lengths, atomic areas, and atomic volumes. Besides being compound relations, these are also wholes, since they have novel properties. Thus out of these separators we get a geometry on a nominal orthogonal lattice. Angles other than right angles might emerge with Diophantine right-angled triangles, in which the emergent hypotenuse has a length which is an integral multiple of an atomic length, and in which there are emergent angles between the hypotenuse and the other two sides of the triangle. Clearly, as the length of a linear separator decreases towards the limit of zero, this macroscopic geometry approaches the limit of a Euclidean geometry on the continuum; but the limit itself has exclusively nominal meaning.

\* \* \*

When it comes to applied mathematics — specifically, physics — the need to figure out the number of kinds of real prime relation, and their properties and terms, and hence a fundamental theory of physics, is, needless to say, a wonderful opportunity for fame and fortune for some genius. What is offered here is a very pale shadow of what is needed, but it nonetheless serves to illustrate how

the concept of prime relation can lead to cascading emergence and to an applied mathematics.

In this pale shadow two more kinds of separator are supposed.

**Def. 5.22**      A **temporal separator** has unit duration, and is asymmetric. It is an **atomic duration**.

**Def. 5.23**      An **atomic vector** has length, like a linear separator, but a length slightly less than that of a linear separator; so a structure of them forming an atomic volume has unit mass, through stressing space according to an inverse square law. An atomic vector has a sense and so is asymmetric.

With our geometry, temporal separators will produce a four-dimensional space-time for applied mathematics. A maximal space will exist for one atomic duration and then be replaced by another, which also exists for one atomic duration; thus each maximal space temporally separates its predecessor from its successor. If the unit length and the unit time are very small — a Planck length and a Planck time<sup>11</sup>, perhaps — the geometry will closely approximate a continuous one at the level of atomic nuclei; the presence of atomic vectors will impose a curvature in the space-time, through stressing linear separators according to an inverse square law, which could close the space-time and have a relativistic space-time emergent out of this absolute space and time; there will be a maximum continuous velocity — one atomic length per atomic duration — equal to the

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<sup>11</sup>The Planck length is  $(G\hbar/c^3)^{1/2}$ , where  $G$  is the gravitational constant,  $\hbar$  is Planck's constant, and  $c$  is the velocity of light. The Planck time is  $(G\hbar/c^5)^{1/2}$ . They are quite small:  $1.6 \times 10^{-35}$  m and  $5.4 \times 10^{-44}$  s, approximately.

velocity of light<sup>12</sup>; the series of temporal separators could form a closed loop, so that the Big Bang and the Big Crunch are the one identical event; and with mass, length, and time we have the basic units of dimensional analysis in physics. The concept of *field* can also be accounted for, in terms of a later concept called *hekerger*. The inadequacies of this pale shadow are obvious: most notably, the properties of prime relations should be such that all of quantum physics, including quantum gravity, is deducible from them. (I say nothing of the seemingly radical probabilistic basis of quantum mechanics, since I share with Einstein the philosophic conviction that the Good Lord does not play dice (see Chapter 8).)

Two other concepts relevant to applied mathematics may also be defined here:

**Def. 5.24**      A **boundary** is a compound relation, a set of contiguous dissimilarities.

**Def. 5.25**      A **change** or **event** is a compound relation, a dissimilarity compounded with a duration.

We now have in principle an applied mathematics. A properly designed set of separators would yield a variety of wholes (Def. 4.4) which would be leptons and quarks, or perhaps wholes at a lower level — whatever the physics of the situation would require. Then relations between these would lead to cascading emergence of higher and higher wholes, such as hadrons, nuclei, atoms, molecules, structures of molecules such as living cells, structures of cells such as plants and animals, structures of these such as societies and ecosystems, and a still larger structure, the biosphere. If some structure in one maximal space necessitates a near similar structure

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<sup>12</sup>A velocity of one Planck length per one Planck time is  $(\text{Gh}/c^3)^{1/2}/(\text{Gh}/c^5)^{1/2} = c$ .

in its successor than that necessitation is a causation; a sequence of causations is a process, and the temporal pattern of the process may be represented by an intensional function or a differential equation. At each level there would be novel emergent relations having emergent properties, including, at the level of individual animals, minds of mathematicians and scientists, and, at the level of societies, mathematical and scientific libraries, observatories, and laboratories — all of which are necessary conditions for the existence of mathematics and science as disciplines.

\* \* \*

Turning next to fuzzy set theory and fuzzy logic, we note that the least satisfactory parts of these subjects are their fuzzy foundations: the lack of clear understanding of the concepts of *degree of set-membership*, *degree of truth*, and *degree of inference or validity*. All of these can be given intensional meaning quite easily, as follows.

We begin by extending the concepts of similarity and dissimilarity between two relations from two-valued to multi-valued concepts. We take one of the relations and count the number,  $s$ , of properties that it has which are similar to a property in the other, and the number,  $d$ , of properties which are dissimilar; and if one of the relations has more properties than the other, the number of the excess is  $a$ . Then:

**Def. 5.26**      The **degree of similarity**,  $n$ , between two kinds of relations  $A$  and  $B$ , symbolised  $A \text{t}^n B$ , is the ratio  $n = s/(s+d+a)$ .

For example, suppose that we want to compare triangles and squares in Euclidean geometry. We may treat each of them as a simple, closed, figure which has an adicity, which is the number of sides that it has; and which has two emergent properties: area, and shape. These can be described by the lengths and ordering of some

or all of the sides, and the angles between them. So they all have five properties: simple figure, closed figure, adicity, area, and shape. Any two geometrically similar triangles have all five properties similar, and hence a degree of similarity of one. Any two congruent triangles of unequal area have four similar properties, and hence a degree of similarity of 4/5. A triangle and a square of equal area differ in their adicity and shape and so have a degree of similarity of 3/5. And a triangle and a square of unequal area differ in their adicity, area, and shape and so have a degree of similarity of 2/5.

- Def. 5.27** The **degree of dissimilarity**,  $m$ , between two kinds of relations A and B, symbolised  $AT^mB$ , when  $At^nB$ , is  $m = (d+a)/(s+d+a) = 1-n$ .
- Def. 5.28** If  $At^nB$  then the **degree of membership**,  $l^n$ , of A in the similarity set  $\{A(tB)\}$  is  $n$ . This is symbolised by  $Al^n\{A(tB)\}$ .
- Def. 5.29** If  $At^nB$  then the **degree of non-membership**,  $L^m$ , of A in  $\{A(tB)\}$  is  $m = (d+a)/(s+d+a) = 1-n$ . This is symbolised by  $AL^m\{A(tB)\}$ .
- Def. 5.30** A **fuzzy set**, of degree of fuzziness  $n$ , is a similarity set defined by a degree of similarity  $n$  to a property set P:  $\{A(t^xP)\}$ , for all  $x$  such that  $n \leq x \leq 1$ .
- Def. 5.31** If A is a representation or copy of B and  $At^nB$  then the **degree of truth**,  $u^n$ , of A, relative to B, is  $n$ . This is symbolised  $u^nA$  if A is an abstract idea and B is a corresponding part of reality.
- Def. 5.32** If  $At^nB$  then the **degree of falsity** of A relative to B,  $U^m$ , is  $m = (d+a)/(s+d+a) = 1-n$ .

Since intensional inference is based on necessity, or singular degree of possibility, intensional degree of inference must be based on plural degree of possibility, or degree of contingency. Suppose that  $P \rightarrow Q$ , that  $P$  has  $p$  properties and  $Q$  has  $q$  properties, the degree of truth of  $P$  is  $n$ , and the degree of truth of  $Q$  is  $m$ . We know already that if  $n=1$  then necessarily  $m=1$ , and if  $m<1$  then necessarily  $n<1$ : these are *modus ponens* and *modus tollens*, valid inferences whose “degree of validity” might be described as unity because the degree of possibility between the truth values of the antecedent and consequent is one. So if  $n<1$ , we may ask what the degree of possibility of some specific value of  $m$  is; and, clearly, this is a probability depending on  $p$ ,  $q$ , and  $n$ . Equally, we may ask for the probability of a specific value of  $n$ , given a value of  $m$ ; depending upon the circumstances, we would be enquiring into *modus tollens*, the fallacy of affirmation of the consequent, or induction. Thus intensional degree of inference, or degree of validity, is a matter of probability. This should not be confused with nominal degree of validity, which is truth-functional.

A word of caution is needed here, however: it may well be that relations such as degree of membership are purely nominal, in which case there is no intensional fuzzy set theory or intensional fuzzy logic.

Finally in this chapter we may utilise the definition of degree of similarity to define the degree of similarity between two sets and hence between two wholes.

**Def. 5.33** Given two sets,  $A$  and  $B$ ,  $A$  smaller than  $B$ , and any ordering of  $A$ , then each member in this ordering may be matched with a member of  $B$ , such that no member of  $B$  is matched more than once. This matching thus produces a contingent function from  $A$  to  $B$ , and between each argument of this function and its value is a degree of similarity. The average of these degrees of similarity for every argument of the function is a ratio,  $r$ , say. There is then some intensional function from  $A$

to  $B$  that maximises  $r$ . Any pair of an argument of this intensional function and its value is defined as a pair of **corresponding members** in the two sets, and the maximal value of  $r$  is the **degree of similarity** between the two sets.

Since a whole is a set of elements which are relations, the degree of similarity between any two wholes is defined. In practice it is usually necessary to go down only one or two levels in comparing wholes. Also, in everyday affairs we usually revert to two-valued comparisons by having some fairly high cut-off value of degree of similarity, such that anything above this value is called similar and anything below is called dissimilar. For example, we call apples dissimilar to oranges, while we call two apples similar, and two oranges similar, even though they are not perfectly similar. This also applies to our concept of truth (Def. 4.6).

## 6. Some Theorems

We end Part One with the proofs of eleven elementary theorems in intensional set theory, in order to justify earlier claims.

We begin with the primitive concept of relation, as characterised in Chapter 1. We take as an axiom the existence of a sufficient quantity and variety of prime relations to yield, by cascading emergence, a universe of discourse adequate for our purposes — including similarities and dissimilarities, possibility relations, and intensional truths. This universe is the set of all genuine relations, which may be treated as a set of property sets. Remember, however, that couplings and commonalities of property sets are not necessarily members of this universe: they are members only if relations having such property sets exist. We also assume the definitions of the intensional connectives and of the extensional connectives, and claim that they are the intensional and extensional meanings of the grammatical connectives; and we claim that the basic argument forms of mathematics are valid. And, finally, we note that in the use of the expression  $\{AA\}k\{AB\}$ , the disjunction of subset and identity in the symbol ‘k’ is a complete disjunction, so that disjunctive syllogism may be applied to it validly.

**Theorem 1.** For any intension,  $A$ ,  $M\{AA\}tA$ . (6.1)

*Proof.* For each  $x$ ,  $x\{AA\}qxA$ ,  $B M\{AA\}tA$ .  $\square$

**Theorem 2.** For any set  $S$ , if  $MS$  exists then  $Sk\{A(MS)\}$ . (6.2)

*Proof.* Assume that  $MS$  exists.  $(x\{S\}q(x(MS)))$  and  $(x(MS))q(x\{A(MS)\})$ , so  $(x\{S\}q(x\{A(MS)\}))$ .  $B Sk\{A(MS)\}$ , by definition.  $\square$

**Theorem 3.**  $(S=\{A(MS)\})S(S \text{ is an intensional set}).$  (6.3)

*Proof.* (i) Suppose that  $MS$  exists and  $S=\{A(MS)\}$ : then  $S$  is an intensional set, since  $\{A(MS)\}$  is an intensional set. (ii) Suppose that  $S$  is an intensional set,  $\{AS\}$ ; by Theorem 1,  $M\{AS\}tS$ ; therefore  $S=\{AS\}=\{A(M\{AS\})\}=\{A(MS)\}$ , by the principle of substitution of equivalents.  $\square$

**Theorem 4.** An extension  $S$  is a contingent set if and only if either  $MS$  does not exist, or else  $MS$  exists and  $Si\{A(MS)\}$ . (6.4)

*Proof.* We note that the disjunction is complete, in that to say that  $MS$  neither exists nor does not exist has exclusively nominal meaning. (i) If  $MS$  does not exist then  $S$  cannot be an intensional set, by Theorem 3, so  $S$  is either a nominal set or else a contingent set, and this disjunction is complete in that there does not exist a set that is not intensional nor extensional nor nominal; but  $S$  is not an exclusively nominal set because it is an extension; so  $S$  is a contingent set. (ii)(a) Suppose, using *reductio*, that  $MS$  exists, that  $Si\{A(MS)\}$ , and that  $S$  is not a contingent set;  $S$  is not an exclusively nominal set, since  $MS$  exists, so  $S$  is a necessary set, in which case  $S=\{A(MS)\}$ , by Theorem 3, which contradicts our assumption; so  $S$  is a contingent set. (b) Suppose that  $S$  is a contingent set, hence not a necessary set;  $Sk\{A(MS)\}$ , by Theorem 2, which means that either  $S=\{A(MS)\}$  or  $Si\{A(MS)\}$ ; but if  $S=\{A(MS)\}$  then  $S$  is a necessary set, by Theorem 3; so  $Si\{A(MS)\}$ .  $\square$

**Theorem 5, the equivalence theorem:**  $(AtB)s(\{AA\}=\{AB\})$  (6.5)

*Proof.*  $(\{AA\}=\{AB\}) \text{ s } (xl\{AA\}sx\{AB\}) \text{ s } (xAsxB) \text{ s } (AtB) \square$

**Corollary.**  $(ATB)s(\{AA\} \neq \{AB\}).$

**Theorem 6, the implication theorem:**  $(A \circ B) \subseteq (\{A\} \cap \{B\})$ . (6.6)

*Proof.*

(a)  $(A \circ B) \subseteq (x \in A \rightarrow x \in B) \subseteq (x \in \{A\} \rightarrow x \in \{B\}) \subseteq (\{A\} \subseteq \{B\})$ , by Theorem 1; but  $(A \circ B) \subseteq (A \subseteq B)$ , so  $\{A\} \subseteq \{B\}$ , by Theorem 5, Coroll.  $B \subseteq A$   $(A \circ B) \subseteq (\{A\} \cap \{B\})$ .

(b) The converse is proved similarly.  $\square$

**Theorem 7, the conjunction theorem:**  $\{A \cap B\} = \{A\} \cap \{B\}$ . (6.7)

*Proof.*  $x \in \{A \cap B\} \iff x \in (A \cap B) \iff (x \in A \text{ \& } x \in B)$  by definition of coupling. But  $x \in A \iff x \in \{A\}$  &  $x \in B \iff x \in \{B\}$ . So  $x \in \{A \cap B\} \iff (x \in \{A\} \text{ \& } x \in \{B\}) \iff x \in (\{A\} \cap \{B\})$ , by definition of intersection.  $\square$

**Corollary**  $(A \cap B) \subseteq \{A\} \cap \{B\}$ .

**Theorem 8.** For all intensional sets  $A, B$ ,  $(M(A \cap B) \subseteq (M(A) \cap M(B)))$ . (6.8)

*Proof.* Suppose that  $A = \{A_1, \dots, A_n\}$  and  $B = \{B_1, \dots, B_m\}$ ; then  $A \cap B = \{A_1, \dots, A_n, B_1, \dots, B_m\}$ .  $B \subseteq (A \cap B) \subseteq (A_1 \cap \dots \cap A_n \cap B_1 \cap \dots \cap B_m) \subseteq ((M(A) \cap M(B)))$ .  $\square$

**Theorem 9, the disjunction theorem:**  $\{A \cup B\} \subseteq \{A\} \cup \{B\}$ . (6.9)

*Proof.* Because  $\{A\}$  and  $\{B\}$  are intensional sets,  $M(\{A\} \cup \{B\})$  exists, by Theorem 8. So  $M(\{A\} \cup \{B\}) \subseteq (M\{A\} \cup M\{B\}) \subseteq (A \cup B)$ , by Theorems 8 and 1; but  $\{A\} \cup \{B\} \subseteq M(\{A\} \cup \{B\})$ , by Theorem 2.  $B \subseteq \{A \cup B\} \subseteq \{A\} \cup \{B\}$ .  $\square$

**Corollary.**  $(\{A(AmB)\}h\{AA\}g\{AB\})s(\{AA\}g\{AB\})$  is a contingent set),  
and  $(\{A(AmB)\}= \{AA\}g\{AB\})s(\{AA\}g\{AB\})$  is a necessary set).

**Theorem 10.** (i) If  $Av(C_1|C_2|...C_n)$  then  $(C_1mC_2m...C_n)tA$  and  $C_1mC_2m...C_n$  is a complete disjunction.

(ii) If  $Av(C_1|C_2|...C_n)$ , the commonality of any subset,  $S$ , of  $\{C_1, C_2, ...C_n\}$  is an incomplete disjunction. (6.10)

*Proof.* (i)  $Av(C_1|C_2|...C_n)$  is more properly written in the form  $Av((C_1nA)|(C_2nA)|...(C_nnA))$ . By the definition of a possibility relation (Def. 1.12), any two consequents of  $A$ ,  $C_hnA$  and  $C_jnA$ , are mutually exclusive, hence  $(C_hnA)m(C_jnA)tA$ ; so  $((C_1nA)m(C_2nA)m...(C_nnA))tA$ . Also by Def. 1.12, the consequents of  $A$  are exhaustive; so  $\{AA\}=(\{A(C_1nA)\}g\{A(C_2nA)\}g...\{A(C_nnA)\})$ ; thus  $\{AA\}$  is an intensional set, by Th. 3, and so is a complete disjunction, by Def. 3.13.

(ii) If we take the union,  $V$ , of the extensions of any subset,  $S$ , of the  $n$  consequents of  $A$ ,  $S=\{C_hnA, ...C_jnA\}$ ,  $V=\{A(C_hnA)\}g...\{A(C_jnA)\}$ , then because any two consequents of  $A$  are mutually exclusive, it follows that  $(MV)tA$ , in which case  $Vi(MV)$  and  $S$  is an incomplete disjunction.  $\square$

**Corollary.**  $(C_1-A), (C_2-A), ... (C_n-A)$  are all disparate.

**Theorem 11, the negation theorem:** if  $P$  and  $P'$  exist then  $\{AP\}'=\{AP'\}$  and  $\{AP'\}'=\{AP\}$ . (6.11)

*Proof.* If  $P$  and  $P'$  exist and  $Av(P|P')$  then  $At(PmP')$ , by Theorem 10. So, by Theorem 9,  $\{AA\}=\{A(PmP')\}=\{AP\}g\{AP'\}$ , since the disjunction is complete. Hence  $\{AP\}'=\{AA\}-\{AP\}=\{AP\}g\{AP'\}-\{AP\}=\{AP'\}$  and, similarly,  $\{AP'\}'=\{AP\}$ .  $\square$



## PART TWO

### Intensional Philosophy of Mathematics

#### *Preface*

A purely intensional mathematics has two merits: axiom generosity, and freedom from paradox. We will rely on both of these in the following solutions of problems in philosophy of mathematics.

The main problems in philosophy of mathematics are:

1. What is it that determines mathematical inferences, as if there were a separate Platonic reality which mathematics describes but cannot alter?

2. Why are axiom sets in mathematics so rich in consequences, so generous with theorems?

3. What is the explanation of both the power and the beauty of mathematics, each of which puts mathematics in an entirely different category from all other languages?

4. What is the difference between mathematical discovery and mathematical invention? And, whichever is involved, what is the origin of mathematical novelty?

5. Why is mathematics so effective in describing the world?

6. Given our distinction between intensional, extensional, and nominal mathematics, what is the extent of intensional mathematics?

The first two of these have already been solved: mathematical inferences are based on relations of necessity, or extrinsic properties that include necessity, and axiom generosity is due to cascading emergence of relations.

The fifth problem is properly a problem in philosophy of science, but a brief answer can be offered here: mathematics is effective

in describing the world because reality is relational and mathematics is our language of relations. The close relationship between mathematics and reality will be discussed in Chapter 8.

So we are left with the third, fourth, and sixth problems. We consider the problems of explaining the power and beauty of mathematics, and its novelties, in Chapter 7, and leave the sixth problem to Chapter 8.

For explaining the power, beauty, and novelties of mathematics we need a concept called *hekergy*<sup>13</sup>.

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<sup>13</sup> This word was invented for me by Prof David Gallop, a classicist, who was then one of my teachers of philosophy at the University of Toronto.

## 7. Hekergy

The concept of hekergy comes from a generalisation of the concept of negative entropy<sup>14</sup>.

\* \* \*

Certain compound relations have probabilities. If a compound relation  $C$  has a term set  $C$ , then  $C$  may give rise to a variety of possible compound relations:  $CV(C_1|C_2|\dots C_t)$ , where  $t$  is the total number of the variety. The probability (Def. 1.18) of  $C$ , given  $C$ , is then  $1/t$ . And if there is some criterion  $R$  that determines a subset of this variety,  $RV(C_i|C_j|\dots C_e)$ , where  $e$  is the number of the variety of the subset, then the probability of  $R$ , given  $C$ , is  $e/t$ .

An important case is the probability of an arrangement (Def. 4.3). If  $R$  is a top, or unifying, relation of a whole,  $W$ ,  $R$  is the set of its terms, and  $A$  is an arrangement of  $R$ , then  $RV(A_1|A_2|\dots A_t)$ , where  $t$  is the total number of possible arrangements of  $R$ , and  $RV(A_i|A_j|\dots A_e)$ , where  $e$  is the number of arrangements of  $R$  in which  $R$  emerges. Thus the **probability** of the whole,  $W$ , given  $R$ , is  $e/t$ .

**Def. 7.1** If a relation  $R$  emerges with the number  $e$  of arrangements of its term set  $R$ , and there are  $t$  possible arrangements of its terms altogether, so that the probability of the whole defined by  $R$ , given  $R$ , is the ratio  $e/t$ , then the **improbability of the whole** is the ratio  $t/e$ .

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<sup>14</sup>The term negative entropy is a misnomer: for entropy to be negative requires a temperature less than absolute zero, which is impossible. The name should be *negated* entropy: entropy with a minus sign in front of it. However the incorrect usage seems to have become established.

We note that, having denied infinite intensional natural numbers, neither  $t$  nor  $e$  is ever infinite; thus the probability and improbability of a compound relation are always defined. The letters  $e$  and  $t$  stand for the numbers of *equivalent arrangements* and of *total arrangements*; the equivalent arrangements are equivalent in making  $R$  emerge.

**Def. 7.2**      A measure of the **entropy**,  $S_R$ , of a relation  $R$  is the natural logarithm of the probability of  $R$ ,  $\ln(e/t)$ ; and a measure of the **hekergy**,  $H_R$ , of  $R$  is the natural logarithm of the improbability of  $R$ ,  $\ln(t/e)$ .

Hekergy, and hence its negation, entropy, is a property of every relation which has a probability. The logarithms of the probabilities are used because probabilities combine multiplicatively while hekergies combine additively.

The definition of entropy given here is, of course, a generalisation of the entropy of physics. In physics a macrostate may be thought of as an emergent relation unifying a whole, and a microstate as any arrangement of molecules that causes that macrostate to emerge — so that  $e$  is the number of microstates that produce that macrostate. Physicists use the number  $W$ , called the thermodynamic probability, for this  $e$ , and define the entropy  $S$  as  $S = k \cdot \ln W$ , where  $k$  is Boltzmann's constant, which gives the entropy its dimensions of heat per degree of temperature. The number  $t$  is not used in physics: it is an additive constant that can be ignored in a closed energy system; because of this  $W$  is not a probability in the mathematical sense of the word.

The total hekergy of a whole consists of the hekergy of its unifying relation plus the total hekergies of all of its terms. When the distinction is needed, these will be called:

**Def. 7.3**      The **emergent hekergy** of a whole is the hekergy of its top relation.

**Def. 7.4**      The **summation hekergy** of a whole is the sum of the total hekergies of all of the terms, and subordinate terms, of its unifying relation.

The summation hekergy thus includes the hekergies of all the subordinate parts of the whole.

In the expression “The whole is greater than the sum of the parts” the greatness of the sum of the parts is the summation hekergy and the excess of the whole over the sum of the parts is the emergent hekergy.

Although it will seem implausible at first, we here state, and will demonstrate in the next chapter:

**The Principle of Conservation of Hekergy**  
is the principle that hekergy is neither created nor destroyed, it is always conserved.

This will seem implausible because it appears to contradict the well established second law of thermodynamics, which gives high probability to entropy increases and low probability to entropy decreases, so that, overall, total entropy increases. This law is established, however, by measurements only within the realms of physics, chemistry, and information theory, and then only within ideally closed energy systems, which are entirely fictitious. In other realms it is common for order to emerge out of chaos, and such order emerging out of nothing is unexplained entropy decrease. That is, this so called order is a relation, and its emergence is its coming into existence, which thereby entails a hekergy increase. In terms of hekergy, thermodynamics requires overall hekergy decrease, while allowing temporary hekergy increases; and it is here claimed that all of these decreases and increases are equal and opposite, so that, overall, hekergy is conserved. The most notable example of hekergy increase over time is biological increase, as organisms evolve into ever more

complex, and thereby improbable, life forms. Textbooks on statistical mechanics usually take note of this fact, but add that such entropy decreases occur only as a result of greater increases elsewhere, such as in the Sun. But these “greater” increases are unproved, mere dogma. Furthermore, such entropy decreases are normally ignored in thermodynamics: they are neither measured nor calculated — because thermodynamics confines itself to low level systems.

Historically, physics began to integrate with biology when Erwin Schrödinger<sup>15</sup> defined life as very high negative entropy in dynamic equilibrium.

Significant in this context is the fact that as the Universe expanded and cooled, following the Big Bang, higher level systems gradually emerged successively: galaxies, stars with planets, life, multicellular organisms, societies, and ecosystems. Each new level constituted significant hekerger increase.

One significance of biological hekerger increase is that every living organism will have an intrinsic need to increase hekerger; or, when this is not possible, to preserve it; or, when this is not possible, to minimize the decrease. Consequently, in organisms as complex as humans there must be two capabilities: one of recognising hekergeries, and another of cherishing them. It is here claimed that we call them **human values**, and traditionally distinguish three kinds: truth, beauty, and goodness. We are also familiar with the operation of the second law of thermodynamics, in that values generally are hard to gain but easily lost; and we also encounter unexplained increases in value in such things as mathematical intuition and discovery, and scientific invention of theories and design of experiments.

Let us briefly consider truth, beauty, and goodness.

We have encountered truth already, in the form of intensional, or similarity, truth: if a kind of a relation is similar to a kind of relation in reality then it is intensionally true, and it is otherwise false (Def. 4.6).

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<sup>15</sup>Erwin Schrödinger, *What is Life?* Cambridge, 1944.

We have already defined (Def. 5.31) the degree of truth of a copy, relative to its original as the degree of similarity between them. Since this has an improbability, such truth has a hekergy. Clearly, if a copy is large and perfectly true then its hekergy is high, and as the number of departures from perfect truth increases, the hekergy diminishes rapidly.

Beauty may be expected when relations of high hekergy emerge, and this is particularly likely when there is cascading emergence, as in rich mathematical axiom systems. Thus mathematical beauty is explained, in principle, at least. For example, when, in our undergraduate years, we first meet the expression  $e^{i\pi} = -1$  we are amazed because this relationship between  $e$ ,  $i$ , and  $\pi$  seems so improbable; the improbability is the basis of the hekergy of the expression, which we experience as mathematical beauty.

Goodness, as a concept, belongs in philosophy of mind (see Part Four); but here we may remark that there are basically two kinds of goodness in the human scene: goodness for the self and goodness for society. For a mathematician, mathematical invention and discovery are emergence of new relations, within the mind, and so are personal hekergy increase, or goodness for the self; and publication of such discoveries is social hekergy increase, or goodness for society. And similarly for scientists.

Concerning discovery and invention in mathematics, it is here simply claimed that axiom sets (Def. 8.4) are invented, in the sense of inventing primitive concepts and coupling them into propositions; and the emergents out of axiom sets — theorems — are discovered.

We turn now to the origin of novelty in mathematics.

As was earlier pointed out, relational properties emerge with their relations, so it is quite meaningful to speak of the emergence of a property. Also, not all properties emerge at the lowest structure level: different properties emerge for the first time at different levels.

**Def. 7.5**      The **emergent level** of an intrinsic relational property is the lowest level at which that property may emerge.

Properties may emerge at levels higher than their emergent levels, but not lower. For example, a simple machine may be defined as a machine which has no machines as parts, while a complex machine does have machines as parts. A mechanical clock is a simple machine; and an automobile is a complex machine, having machines such as motor and transmission as parts. The clock is at the emergent level of the emergent relation *working order*, while the *working order* of an automobile is at a higher level. Another example is life, which emerges at either the viral level or the cellular level, and also at the multicellular level. A multicellular animal may die while some its individual cells continue to live for a while, and *vice versa*, which shows that the two levels of life are distinct.

Clearly, at the emergent level of a property, that property is novel, in the sense that it does not occur at any lower level. Thus as one metaphorically travels up through the levels of a system, novelties emerge.

### **The Principle of Novel Emergence**

is the principle that, with some exceptions, every emergent relation has an emergent level and that, at that level, it possesses at least one property not possessed by its terms, or by any of its subordinate terms.

Excluded from this principle are prime relations, which, being at the lowest level, do not have novel properties, relative to lower levels. Similarities and dissimilarities and other compoundable relations are also excluded, and so are set relations. Recall that a whole is unified by a relation having a novel property, unlike a compound relation and an intensional set, which are unified by relations not having novel properties.

In a good mathematical system there is cascading emergence, and hence plenty of novelty.

\* \* \*

One further point arises from the concept of hekerger, in connection with our earlier pale shadow of an applied mathematics. A **field** might be defined as a map of how the associated hekerger changes must occur. When charges, masses, etc., move according to the dictates of a field, this movement is a rearrangement of the terms of a relation and thereby determines either the change or the conservation of the hekerger of that relation.

## 8. The Ontological Argument

Our next concern is the extent of intensional mathematics: what intensional mathematical entities exist? The ontological argument is the argument that the intensional mathematical system that exists is the best of all possible mathematical systems, and that it exists necessarily because it is the best. This argument relies on the two strengths of intensional meaning, axiom generosity and freedom from paradox, in that *best* is defined by means of axiom generosity and *existence* by means of logical consistency.

**Def. 8.1**      Intensionally, a **mathematical entity** is any intensional meaning.

So a mathematical entity is a relation, or a property of a relation; this includes intensional sets both in the form of their intensions and in the form of set relations, which latter are their relational meanings, and also compound relations in the form of their uniting relations, and wholes in the form of top relations having novel properties. Note that, as a mathematical entity, an intrinsic property of a relation exists only when it is possessed by an existing relation.

The usual mathematical concept of existence is simply *possibility* or *compatibility*: a mathematical entity exists if it is possible. Such possibility is of two kinds, intrinsic and extrinsic. A mathematical entity is intrinsically possible, or intrinsically compatible, if it is so-called self-consistent: it neither contains nor implies a contradiction; and it is extrinsically possible, or extrinsically compatible, relative to some other entity, if the two together are consistent: they may be related without contradiction. That is, in order to exist a mathematical entity must be intrinsically consistent, and extrinsically consistent with whatever else exists. So if X is defined in a mathematical system it either produces a contradiction or not, so that it either exists within that

mathematical system or not. It produces a contradiction, in being there, either because it is self-contradictory, or else because it is incompatible with other parts of the system: because it is either intrinsically or extrinsically inconsistent. Although this is all well known, it is worth stating it as a principle:

### **The Existential Principle of Mathematics:**

If a mathematical entity A exists, and a mathematical entity B is self-consistent and compatible with A, then B exists; that is, B exists if B is both intrinsically possible, and extrinsically possible relative to A. Conversely, if B is either not self-consistent, or incompatible with A, B does not exist.

In order to be quite clear that the compatibility intended here is intensional compatibility, we define:

**Def. 8.2** Two relations are **extrinsically possible**, relative to each other, or **intensionally compatible**, if both of them could be terms of one relation.

As a philosopher might say: intensionally, to be is to be related.

If a relation is not self-consistent then of course it is a purely nominal relation: it does not exist except as a name and as an analogical meaning in a mind.

**Def. 8.3** Two mathematical symbols, words, sentences, or systems are **nominally compatible** if they do not produce a contradiction when combined.

If two purely nominal relations may be combined without contradiction then they have nominal compatibility but no intensional compatibility. For example, two purely nominal relations are *term of*, supposedly holding between every relation and each of its terms, and the monadic relation of *self-similarity*; and the statement that every

relation *term of* is self-similar is free from contradiction; so the two purely nominal relations are nominally compatible, but they have no intensional compatibility because they do not exist, because of extravagant multiplication. We note that intensional compatibility implies nominal compatibility; so a contradiction, which is a sufficient condition for absence of nominal compatibility, is a sufficient condition for the absence of intensional compatibility; and this applies both to intrinsic and extrinsic possibility.

Thus if a relation  $R$  exists then it is intensionally intrinsically compatible, all of its terms are extrinsically intensionally compatible with each other and with  $R$ , and each term is intensionally intrinsically compatible.

We next ask whether existence, which is possibility, may be necessary existence, which is singular possibility. Since there are two kinds of existence, intrinsic and extrinsic, we can ask about intrinsic necessary existence and extrinsic necessary existence. We have already encountered **extrinsic necessary existence**, in the form of distributive existence: if a relation exists then all of its terms exist necessarily, as well as all of its subordinate terms, down to the necessarily existent prime relations. Extrinsic necessary existence may also be compositional, in the form of emergence: if a term set  $R$  and an arrangement  $A$  exist, and  $A$  is such that  $R$  emerges, then  $R$  emerges necessarily. Distributive and compositional existence also apply to intrinsic properties, since the existence of an instance is necessarily the existence of a kind.

Note that the necessity in distributive and compositional existence is not a relation of singular possibility — if it were there would be an infinite regress — so it is an extrinsic property. Thus if  $aRb$  and  $R$  exists then  $a$  and  $b$  each have the extrinsic property of extrinsic necessary existence; on the other hand, if  $aOb$  and hence  $(ua)q(ub)$  then the necessity,  $q$ , is a relation between the first truth and the second truth — that is, a relation between two relations.

Note also that although the top relation of a whole extrinsically necessitates the existence of its terms, and thereby of all of its

subordinate terms, it does not extrinsically necessitate its arrangement or subordinate arrangements — unless it has maximum hekerger. The number of arrangements that necessitate the emergence of the relation is  $e$ , which is unity for maximum hekerger; but if  $e > 1$  then there is a plural possibility of arrangements, given the existence of the top relation, hence no necessity.

If **intrinsic necessary existence** is possible then it must be a property of a relation such that whatever relation possesses this property has to exist. We offer:

**The intrinsic necessary existence conjecture:**

intrinsic necessary existence is not a self-contradictory concept, so at least one among all possible relations must possess it and thereby exist necessarily.

Until someone demonstrates that intrinsic necessary existence is self-contradictory, this conjecture may be assumed to be true.

What is usually called an axiom set is here more correctly called an axiom structure:

**Def. 8.4**      An **axiom structure** is any structure, or arrangement, of relations that produces cascading emergence, hence has axiom generosity.

**Def. 8.5**      The **emergent structure** of an axiom structure is the complete set of relations cascadingly emergent from it.

**Def. 8.6**      An axiom structure and its emergent structure together are an **intensional mathematical system**, or **mathematical system** for short.

As we have seen, no mathematical system may have an infinite regress of lower and lower levels, since infinity has no intensional meaning: every mathematical system must have lowest level relations,

which are called prime relations, and these relate each other so that each is both relation and term.

**Def. 8.7**      A **prime axiom structure** is an axiom structure consisting of prime relations.

Because each member of a prime axiom structure is a prime relation and so equally relation and term, the extension of a prime axiom structure is identical with its arrangement. A description of a prime axiom structure would describe all of the prime relations in it; the relations cascadingly emergent out of this would be described by subsequent theorems. Every axiom structure may in principle be reduced to a prime axiom structure: every intensional mathematical system has a prime axiom structure.

**Def. 8.8**      If a mathematical system **S** has a prime axiom structure **A**, containing  $a$  prime relations, and an emergent structure **E**, out of **A**, of total hekerger  $c$  then the **wealth**,  $w$ , of **S**, is  $w = c/a$ . If one system has greater wealth than another we will say that it is **richer**, or **better**, than the other, which latter is **poorer**, or **worse**.

The wealth of a system is thus a function of the quantity and quality of emergents from the axiom structure: it is a measure of the generosity of the axiom structure.

One can conceive of a mathematical system being enriched by addition of prime relations. As it does so, the rate of increase of its wealth, relative to the rate of increase of the prime axiom structure, will in principle either allow the wealth to increase to infinity, or follow a law of diminishing returns of wealth with increase of prime axiom structure: the wealth will either diverge, to infinity, or else converge to a limit. So for a system of maximum wealth we could have either infinite wealth, or else the maximum wealth of a finite system. But

infinity is intensionally impossible, so the wealthiest possible system is a finite system of finite wealth. Call this wealthiest possible system **G**.

Being the wealthiest, **G** must have the minimum possible axiom structure for this amount of wealth; call this axiom structure **A**.

**G** could have an ultimate top relation, **T** and therefore would have it, since if it did not it would not be the wealthiest system possible; or, alternatively, **T** is possible and by that fact exists. Every ordinate and subordinate term of **T** would exist necessarily, by extrinsic necessary existence, because of the existence of **T**: that is, the existence of **T** would distributively necessitate the existence of every non-compound relation in **G**. **T** would also necessitate the existence of **A**, because only one minimum possible axiom structure can produce **T**, so **A** is a singular possibility, given **T**. And **A** compositionally necessitates the existence of every compound relation in **G**, and thereby every compoundable relation, hence the existence of all the arrangements of all term sets in **G**. So **T** extrinsically necessitates the existence of everything else in **G**.

We pause here to consider the fact that not every hekerger in **G** is a maximum. That there must exist relations of less than maximum hekerger in the wealthiest of all possible systems is due to the fact that they exist so that a greater hekerger may emerge elsewhere; extensive cascading, to a maximum total hekerger, requires less than maximum hekerger at various lower levels. Not only does this fact allow the appearance of contingency at lower levels, but it reduces these degrees of contingency of arrangements to one, it makes them necessary: arrangements are extrinsically necessitated by relations other than the relations whose term sets they arrange. Another way of looking at this is that there is only one possible wealthiest system, so every relation in it must be necessary, a singular possibility. For example, suppose for the sake of argument that the kind of atom having the highest hekerger is carbon, and the kind of structure of carbon atoms having highest hekerger is diamond, rather than graphite or amorphous carbon; if now the best of all possibles required maximum hekerger at each level, then the (non-gravitational) universe would be one big diamond, and thereby

could not contain life, mathematicians and scientists, or mathematical and scientific libraries, or laboratories.

However, if a relation  $R$  exists extrinsically necessarily in  $G$  then so does the term set,  $R$ , of  $R$ ; but unless the hekerger,  $H_R$ , of  $R$  is the maximum possible, so that  $e$  in  $\ln(t/e)$  is one,  $R$  allows a variety of arrangements:  $RV(A_1|A_2|...A_e)$  — the arrangement of  $R$  is contingent, given  $R$ . Each arrangement is possible, therefore exists; but all the  $e$  arrangements are mutually exclusive. The only way of reconciling these two facts is that each arrangement exists within a maximal space for one or more atomic durations: they all exist, but each at a different time. Understood this way, none of them is contingent, each is necessitated by the prime axiom set, which includes atomic durations.

So the existence of everything in  $G$  is extrinsically necessitated by  $T$ . But what about the existence of  $T$ ?

$T$  cannot have extrinsic necessary existence without circularity, but could have intrinsic necessary existence because intrinsic necessary existence is possible and therefore at least one relation must possess it; and if  $T$  could have it but did not,  $G$  would not be the wealthiest possible system. Therefore  $T$  has intrinsic necessary existence and so  $G$  exists necessarily because it is the best of all possible systems.

Suppose now that some other system,  $H$ , existed as well as  $G$ . This  $H$  could not be incompatible with  $G$ , by the existential principle of mathematics, so would have to be compatible with  $G$ . The two could, and therefore<sup>16</sup> would, be related into a single system — their prime axiom systems would be joined into one — and this combined system would be poorer than  $G$ . If this were possible then the new system could, and so would, be enlarged indefinitely, and so be infinite — which is intensionally impossible. Hence  $H$  cannot exist, and so  $G$  exists exclusively, nothing can co-exist with  $G$ . So there is no relation

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<sup>16</sup>That *could* implies *would* in the present context is due to *could* being possibility, which is existence.

outside of  $G$  having intensional necessary existence. Therein lies the justification of Occam's Razor<sup>17</sup>.

If  $G$  were to contain something contingent then that entity would have to be present in the axiom structure of  $G$ , and present there contingently; this would make  $G$  poorer, and so not the best of all possibles. So  $G$  contains nothing contingent.

If some relation within the best system, other than the top relation, were to have intrinsic necessary existence, such a relation would already have extrinsic necessary existence, so the intrinsic necessary existence would be unnecessary, hence contingent in it — and there is nothing contingent in  $G$ . So only  $T$  has intrinsic necessary existence.

The key points in this argument are:

1. Mathematical existence is possibility. Intensionally this means that to be is to be related, with the exception of the top relation of a system.

2. The wealthiest system must be finite, so there is a law of diminishing returns of wealth with increase of prime axiom structure.

3. The wealthiest possible system could have a top relation, and therefore does, since without one it would not be the wealthiest.

4. This top relation extrinsically necessitates the existence of everything else in the system.

5. The top relation could have intrinsic necessary existence, and therefore does, since without it the system would not be the wealthiest. Hence the wealthiest system exists necessarily because it is the wealthiest.

6. If anything else were to co-exist with the wealthiest, the wealthiest would be poorer and so not the wealthiest; hence the

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<sup>17</sup>Occam's Razor, in a mathematical context, may be stated as: do not multiply the entities in the axiom set beyond the necessity of maximising emergents. In a scientific or philosophical context it may be stated as: do not multiply the entities in a theory beyond the necessity of explaining the empirical facts. The ontological argument shows these two statements to be equivalent.

wealthiest exists exclusively. So no other relation outside of the wealthiest may have intrinsic necessary existence.

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Understanding of the nature of  $G$  may be helped by considering the nature of contingent systems. We may suppose an axiom set, and all of its emergents, and then add to the axiom set some whole which produces no further emergents. Such an addition is contingent, in that it could be any one of a large variety of wholes; and it is also contingent in that it is not necessitated by anything else in the system. We can call such a system a **contingent system**,  $C$ , and remark on the following five points concerning it:

1. Addition of a contingent part to the axiom set of any system reduces its wealth,  $w=c/a$ , since it increases  $a$  while  $c$  is unchanged.

2. A contingent system cannot have a top relation, since the existence of a top relation necessitates the existence of everything else in the system and a contingent part is not necessitated. If  $C$  prior to the addition of its contingent part had a top relation,  $W$ , then  $W$  still exists in  $C$  but is no longer a top relation; and  $W$  and  $C$  must be incompatible, since otherwise they could be related by a new top relation; so, since  $C$  exists,  $W$  does not.

3. A contingent system cannot have a property unique among all intrinsically possible systems, since such a property must belong to a top relation: if it belonged to any lower level relation then any other contingent system could possess that relation, contingently, thereby nullifying its uniqueness.

4.  $C$  must be infinite, since there is no end to the number of contingent parts that may be added contingently to the axiom set. So  $C$  cannot be complete.

5.  $C$  cannot be a whole or an intensional set, since each of these has a unifying relation and is complete. So  $C$  must be a contingent set, in which case it has exclusively extensional meaning: it is not an intensional mathematical system.

Each of these five points shows clearly not only that **G** cannot have any contingent parts in its axiom set, but also that every intrinsically possible intensional mathematical system must be a **necessary system**, a system having a top relation. So when considering the wealthiest among all possibles, we need only look at such necessary systems.

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The next question is whether mathematical existence and real existence are one and the same, or not. We have named the actually existent mathematical world **G**; let us call the **real world R**. **R** is all that exists independently of anyone's mind<sup>18</sup> — we can assume that it is that which theoretical scientists try to describe<sup>19</sup>. We can ask if **G** and **R** have anything in common, and if so, how much. There are a total of eight logical possibilities, as is shown by the Venn diagrams in Fig. 8.1, in which shaded areas represent non-existence.

We begin with two indisputable empirical facts: anyone considering this matter is conscious of at least one empirical relation: for example, what they now see is *external* to their head. More generally, mathematics can be applied to the real world, as even the innumerate counting on his fingers knows; and the success of mathematical theoretical science removes all doubt about this. Secondly, there is cascading emergence in reality: wave-particles, atoms, molecules, cells, plants and animals, societies and ecosystems. So **R** and **G** do intersect.

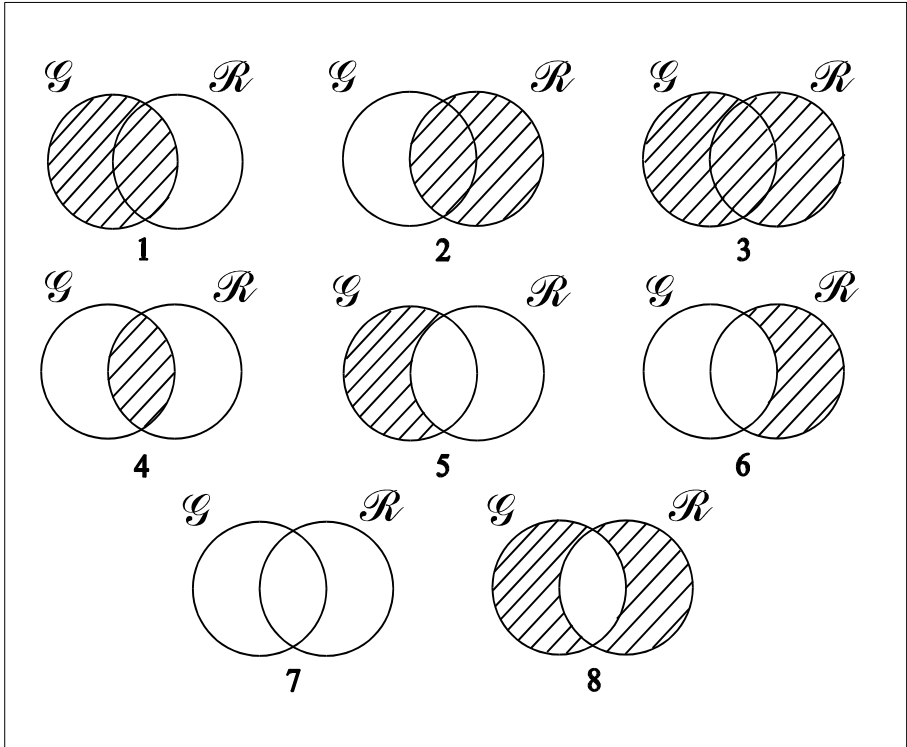
These two empirical facts, together with the mathematical facts that **G** exists necessarily and is complete, require that only one of the

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<sup>18</sup>What is real for person X includes the mind of person Y, but not the mind of X; and what is real for Y includes the mind of X, but not the mind of Y. The real world *per se* includes every mind.

<sup>19</sup>The relation between this and the empirical world we each perceive around us is examined in Chapters 11 and 12.

Venn diagrams can be true: number 8. Diagrams numbered 1 and 3 require  $G$  not to exist; 2 and 3 require  $R$  not to exist; 5 and 7 require that part of  $R$  be added contingently to  $G$ , which is impossible; and 6 requires that  $R$  be a proper subset of  $G$ , which means that  $R$  is incomplete, hence not a possible system — but  $R$  is a possible system because it exists; hence 8 is true:  $G$  and  $R$  are identical, one and the



*Fig. 8.1*

same. And we note also that  $R$  has a definitive property, the property of mind-independent existence, which  $G$  also possesses, since no human mind can contain all of  $G$ .

So the real world exists necessarily because it is the best of all possible worlds.

The keys to this argument are the facts that theoretical science is mathematical, successful, and describes the real world; and that the mathematics of theoretical science is consistent and has axiom generosity. It follows that real existence is intensional mathematical existence.

We note that, given the ontological argument, if intensional mathematics could be developed fully then it would be at once the best possible mathematical system, the ultimate scientific theory from which all lesser theories could be deduced, an *a priori* mathematical metaphysics, and the universal characteristic of which Leibniz dreamed.

There is surprising scientific evidence to support the claim that the Universe, or  $R$ , is the best of all possibles. In the past few decades the subject of cosmology has moved out of philosophy and into physics and astronomy. Edwin Hubble's discovery of the expansion of the Universe led to the necessity of a Big Bang, and then to a detailed description of the early Universe. This description includes the scientific evidence for the Universe being the best of all possibles: namely, the fact of numerous cosmic coincidences<sup>20</sup>. These are that the values and ratios of the strengths of the four fundamental forces, and the values and ratios of the masses of various wave-particles, had to be precisely what they are in order for life to be possible. If they had been different, by even a small amount, then, depending on what was changed, galaxies could not have formed, stars could not have existed, stars could have existed but would have been either too short lived, or else too dim, for life to have evolved on any planet, the known variety of atoms and hence chemical and biological processes could not have existed, or ice would sink rather than float. Any of these would have made the evolutionary process impossible. Some of the precisions involved are extraordinary: for example, if an amount called the density parameter, controlling the rate of expansion of the Universe, was

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<sup>20</sup> See John Gribben and Martin Rees, *Cosmic Coincidences*, Bantam, 1989.

different by one part in  $10^{60}$ , life as we know it could not have evolved<sup>21</sup>.

Each of these coincidences is quite improbable, *a priori*, and the whole set of them vastly more so. So why do they exist?

There are two usual explanations for this. One is by means of the **anthropic principle**<sup>22</sup>: it is supposed that there are a huge number of different mini-universes within one total Universe, such that each mini-universe is causally isolated from every other and has different values for its forces and wave-particle masses — and even, perhaps, different physical laws. In the overwhelming majority of these, life is impossible, but in a few it will be possible. Improbable as the whole set of cosmic coincidences is, in a sufficiently large population of mini-universes, it will occur somewhere. And since we exist, life is possible in our mini-universe, so our mini-universe is one of these improbable ones. This last point is the anthropic principle.

The other usual explanation is that there is only one Universe, without isolated mini-universes within it, and that all of these cosmic coincidences are evidence of design, so that by the argument from design there must be a designer, or God. It is sometimes said that the cosmic coincidences are a fine-tuning of the Universe, so that fine-tuning implies a Tuner.

However, a third explanation is that the Universe exists necessarily because it is the best of all possibles, in which case it must contain life, in which case these cosmic coincidences must obtain. Given this explanation, the other two fail: the first because it multiplies universes far beyond necessity, contrary to Occam's Razor, and the second because design does not imply a designer, given that the best of all possible worlds has that design necessarily.

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<sup>21</sup> *ibid.*, p. 18.

<sup>22</sup> See John Leslie, *Universes*, Routledge, 1989.

It might be thought that the world that we all perceive around us is clearly different from a mathematical system, in that it contains concrete objects and concrete qualities. Since intensional mathematics is wholly abstract, it cannot include the concrete, and so there seems to be a portion of  $\mathbf{R}$  that is not a portion of  $\mathbf{G}$ . The detailed resolution of this problem will be given in Chapter 11, but here it is sufficient to point out that all concrete qualities are illusory: colours, sounds, tastes, smells, and tactile sensations are all secondary qualities, products of our sense organs, mere representations of reality and thereby unreal. The real features of the world we perceive around us are those of empirical science, which are relational, intensionally mathematical.

It might also be thought that because every emergent relation has a probability, *e/t*, it follows that  $\mathbf{R}$  is radically probabilistic. This would be so only if every possible arrangement of the terms of a relation was “equiprobable”, which in turn would require processes in  $\mathbf{R}$  to be stochastic. But in an intensional system this is not possible: the random has exclusively extensional meaning.

Another point concerning the ontological argument is that since the world has maximum possible hekerger, not only does the principle of conservation of hekerger follow, but so do stationary principles. A feature such as a trajectory which has either a maximum or a minimum value has no equivalent arrangements, hence its hekerger is a maximum. Thus the principles of least action, shortest time, etc. are true. Indeed, the ontological argument itself states the ultimate stationary principle.

Finally, concerning the ontological argument, a historical note. The argument was invented in the eleventh century by St. Anselm, who argued that he could conceive of a Being perfect in every respect; such a Being has to exist, since if it did not exist it thereby would be less perfect. By this perfect Being St. Anselm meant God; and we will later see, in Chapter 15, that  $\mathbf{G}$  is one of the possible meanings of the word God. The ontological argument was endorsed by Descartes, Spinoza, and Leibniz, but has been repudiated by most philosophers. The main objection to it is that existence is not a property of things, so that their perfection is unaltered by their emergence or their submergence. The

advantage of the present intensional version of the ontological argument is that it confirms this point for all entities except one: extrinsic necessary existence is not an intrinsic relational property, but intrinsic necessary existence is so. Of all relations only T has intrinsic necessary existence so only T is less perfect in not existing than in existing; all other relations exist extrinsically necessarily because T exists intrinsically necessarily.

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Because of the identity of G and R, we have to identify philosophy of mathematics and philosophy of science, via applied mathematics. Indeed, intensional mathematics is essentially applied mathematics. So our next task is to look at the main problems in philosophy of science.

## PART THREE

### Intensional Philosophy of Science

#### ***9. Problems in Philosophy of Science.***

We first look at a description of science, for clarification of language, and then list seventeen problems in philosophy of science. To begin with we deal only with what might be called intensional science; exclusively extensional science (Def. 12.5) will be considered briefly later.

Science consists of six stages, three of which constitute empirical science and three of which constitute theoretical science. They are:

1. **Data Collection.** The rock on which science is built, without which there would be no science, data collection takes place in the observatory, laboratory, or field. It consists of observations and measurements.

2. **Formulation.** Data may be partially formulated in its collection, in that some data collection is observation or measurement of patterns of data. Otherwise, data is initially formulated chronologically, in note books; later it may be formulated alphabetically, as in an encyclopaedia, or classified, or else formulated mathematically. Classification is the most common formulation in biology, as in taxonomic classification. Mathematical formulation is much the preferred kind because of its success, as in astronomy, physics and chemistry.

3. **Generalisation.** Generalisation is of two kinds, good and bad: good generalisation is scientific, and bad is superstitious, stereotypical, or prejudicial. They are distinguished by the good being

**objective** and the bad being **subjective**. Generalisation, or induction, is the move that what is true so far of experience is true in all similar cases, the move from “Some S are P” to “All S are P”, or from  $(\text{Ex})(\text{Sx}\wedge\text{Px})$  to  $(\text{x})(\text{Sx}\supset\text{Px})$ , a move which is logically invalid — nominally as well as extensionally and intensionally. How to justify objective generalisation is called **the problem of induction**. The product of a scientific generalisation is called a **law**. It is widely believed that laws are empirical, so that they are referred to as empirical laws, but this is incorrect: the empirical is known through the senses and laws cannot be so known, only instances of them can be. Non-empirical knowledge is theoretical knowledge, so that scientific laws are theoretical. Generalisation includes generalisation into the future, so that laws are predictive; this is **prediction of repetition**.

4. **Explanation**. A scientific explanation is a **theory**, which is an axiomatic system or structure; it begins with undefined, or primitive, concepts of imperceptible objects, and unproved propositions, or axioms, concerning them. Explanation is of two kinds: scientific and philosophic. In each a theory is invented to explain one or more laws. If the laws can be deduced within the theory then this is scientific explanation, while philosophic explanation is causal: to describe causes is to explain their effects. The two kinds of explanation are identical if theories in fact describe real causes — as is supposed in the expression that theories describe the **underlying causes** of observed patterns of data. *Underlying* in this context means imperceptible, non-empirical, theoretical.

5. **Prediction of Novelty**. An outstanding feature of theories in the mathematical sciences is their ability to predict empirical novelties, both successfully and often. Many instances are well known, such as the theoretical predictions of radio, gravitational bending of light, anti-particles, nuclear energy, coherent light, holograms, and black holes — all of which were predicted from theories before their empirical discovery. Only the mathematical theoretical sciences can do this. Creation of a good theory requires genius.

**6. Design of Experiments.** In a mature science experiments are designed in order to test theoretical predictions of empirical novelty. If the prediction is successful then the novelty is observed for the first time. Most of the great experiments in the history of science are so elegantly simple that it is easy to overlook the fact that their invention required creativity of genius order. Experiments are a form of data collection and their results verify or falsify the theories that they are designed to test. In an immature science experiments are designed to test conjectures.

**Empirical science** consists of data collection, formulation, and design of experiments; **theoretical science** consists of generalisation, explanation, and prediction of novelty. Cycling repeatedly through the six stages of science is sometimes called **successive approximation to the truth**. This expression illustrates the fact that scientists do not claim truth for laws and theories, but only an approach to truth, called the **probability** of the laws and theories.

Tentative laws and theories are called **hypotheses**.

Empirical science and theoretical science each have their own criteria for what constitutes good science. In empirical science the criteria are (i) objectivity, (ii) quantitative data are better than qualitative data, and (iii) experiments must be repeatable. In theoretical science there are two falsifying criteria and at least seven verifying criteria. The falsifying criteria are: (i) contradiction within a theory and (ii) theoretical contradiction of empirical data; that is, the theory must be both intrinsically possible, and extrinsically possible relative to the empirical data. The verifying criteria are (iii) the size of the scope of the theory, (iv) the density of detail within a given scope, (v) successful prediction of empirical novelty, (vi) integration, or harmony, with other probable theories, (vii) simplicity of theory, (viii) beauty of theory, and (ix) symmetries within the theory. The last four of these need some additional remarks.

Integration of three theories occurred when Maxwell united electricity, magnetism, and optics with the equations named after him; thermodynamics and Newtonian mechanics were integrated by

Boltzmann and Gibbs into statistical mechanics; physics and chemistry were integrated by quantum mechanics; and Erwin Schrödinger began the integration of physics and biology with his definition of life in terms of negative entropy. All of these were strong verification of the theories that were integrated. Simplicity of theory might be defined as the ratio of consequences to hypothesis: the more consequences for the fewer assumptions, the simpler the theory — simplicity thus being the wealth (Def. 8.8) of the theory. Beauty is a concept that was considered in Chapter 7. Symmetries include simple symmetries over time, as with principles of conservation, and the invariance of laws through transformations of co-ordinate systems.

\* \* \*

We consider first philosophical problems of science as a whole, then of empirical science, and then of theoretical science.

1. Why are there two kinds of science, empirical and theoretical?
2. What is the basic relationship between empirical science and theoretical science?
3. What is the relationship between empirical reality (Def. 10.3) and theoretical reality (Def. 10.4)?
4. What is the relationship between laws as generalisations and laws as theorems deduced within theories?
5. What do scientists mean by probability when they speak of laws and theories being probable?
6. Why is science so much more successful than any other human means of gaining knowledge?

In empirical science we have the problems of:

7. What is scientific objectivity?
8. Why do the criteria of empirical science work?

9. The problem of induction: how can induction in science be justified when induction in superstitious, stereotypical, and prejudicial generalisations are so clearly unjustifiable?

In theoretical science we have first of all the problems associated with theoretical entities:

10. Why are theoretical entities all abstract entities?

11. Do they exist in reality, or not?

12. If they exist, where do they exist?

13. Since we cannot have empirical knowledge of them, what kind of knowledge of them do we have?

14. How does theoretical science predict empirical novelties, successfully and often? The importance of this problem is shown by how many there are who cannot make such predictions. Apart from a few exceptions, poets cannot do this, nor astrologers nor soothsayers nor prophets nor think-tanks nor biologists nor social scientists nor empirical scientists.

15. A closely related problem is: why can only mathematical theories do this?

16. Why do the criteria of theoretical science work?

17. Finally, there is the problem of discovering precisely the methods of science.

\* \* \*

There is a simple and elegant solution to all of these problems provided that a theory of observation, or perception, called the Leibniz-Russell theory, is accepted. Indeed, the solution of the problems is strong grounds for accepting this theory, even though such acceptance is difficult. It is difficult for the same reason that, in their day, Copernicus' heliocentric theory and Darwin's evolution were difficult: it is contrary to contemporary common sense and established belief. In the next chapter we discuss various problems of observation, and how they are all special cases of **the general problem of perception**; the

Leibniz-Russell theory, which provides a solution to this problem, is presented in the chapter after that, Chapter 11, and its application to the problems in philosophy of science in Chapter 12.

It cannot be emphasised enough that observation is the fundamental starting point of science, the defining quality of the empirical, yet the common sense view of observation is incompatible with a large number of facts about observation. So a common sense philosophy of science cannot be consistent. This will become perfectly clear in the next two chapters.

## 10. Observation

For clarity we begin with some definitions, all of which should be unproblematic.

**Def. 10.1**      The **empirical** is anything known through the senses, anything observable.

**Def. 10.2**      The **theoretical** is the non-empirical, anything not known through the senses, but which is generally believed to exist.

Examples of the theoretical in everyday life are conscious minds other than one's own and the existence of empirical objects between occasions of their being perceived.

Although all observations are empirical, measurements may be ambiguously empirical or theoretical if their status is not spelled out. A measurement is empirical, but that which is measured is usually not empirical and therefore theoretical. For example, in thermodynamics temperature is defined operationally as that measured by a thermometer, while in statistical mechanics temperature is average kinetic energy of molecules: thermometer readings are empirical, but kinetic energy of molecules is theoretical and, we believe, is also that measured by a thermometer. Or a reading on a voltmeter is empirical but the voltage measured is not. (The voltage must not be confused with a sensation of electric shock that a voltage may produce: the empirical shock is an empirical effect of the theoretical voltage.) On the other hand, a length measured with a yardstick is an empirical length, and the measurement is also empirical; so are a human-liftable weight and the measurement of it.

**Def. 10.3**      **Empirical reality** is all that we perceive around us that is potentially universally public.

We are generally agreed that privacies in perception, such as hallucinations and after images, are unreal. So the real in observation must be public. But some illusions are public to some extent: for everyone in one location the railway lines appear to meet in the distance, even though we all know that such meeting is illusory, and so the illusion is public to all these people; but it is not universally public, since the lines do not meet at that distant point for anyone who is at that point. Hence universality is a necessary criterion. But the universal publicity does not have to be actual: if only one person observes something, in such a way that, if many people were to observe it, it would be public to all of them, then it is potentially public. Thus potential universality of publicity is sufficient.

**Def. 10.4**      **Theoretical reality** is all that exists independently of being observed.

Our definitions so far are ambiguous with regard to the mental: pleasures and pains, feelings and judgments, dreams and fantasies, memories, anticipations, beliefs, etc. We normally classify our knowledge of these as introspective. The problem is: is the mental empirical, or theoretical? If introspection is a kind of perception then one's own mind is empirical, but other people's minds cannot be perceived and so are theoretical. And sometimes the definition of the empirical is extended to include introspection in observation, so that the theoretically real is all that exists independently of consciousness, or even independently of mind. This can be sorted out by stating clearly the empirical-for-whom and the theoretical-for-whom, but for simplicity in the following we will generally exclude introspective data from the discussion

Unless otherwise qualified, the words *real* and *reality* will hereafter refer to the theoretical real.

**Def. 10.5**      **Common sense realism**, or **realism** for short, is the belief that the empirically real is theoretically real.

According to realism, all that we perceive around us which is potentially universally public continues to exist between occasions of being perceived.

**Def. 10.6**      **Empirical perception** is perception as we know it in everyday experience.

Each of us only knows his or her own empirical perception but our experience of it is generally confirmed by reports from other people, so that there is a consensus on the nature of empirical perception. This consensus is:

1. That each of us is *directly conscious*, of *objects*, and of *qualities*, and also of *relations* between these — such as spatial, and temporal relations.
2. The consensus also includes the facts that all of these objects, qualities and relations are *external* to the body of the perceiver.
3. They are also usually *reperceptible*, in that when we return our perception to them they reappear; short-lived things, such as a flash of lightning, are not reperceptible, but these are exceptions that prove the rule.
4. It is also universally characteristic of empirical perception that we each of us perceive from our own *viewpoint*; the location of this viewpoint is the location of our own empirical body, a location that we describe as “I, here, now” and which is the origin of a subjective co-ordinate system whose axes are *in front of me*, *behind me*, *my left*, *my right*, *above me*, *below me*, *my past* and *my future*.
5. What we empirically perceive around us is always bounded by *horizons of the moment*, beyond which we cannot see: the farthest of these are the geographical horizon, and the blue sky on a sunny day or the black sky with stars on a clear night.

6. We do not perceive, but have good grounds for believing, that what is empirically perceived is *material*, as opposed to *mental*.

7. It is also *public* — perceptible by others — as opposed to the mental content of consciousness, which is *private* to each of us.

8. Finally, subsequent to empirical perception is memory: when remembering empirically perceived things we are conscious of mental, and hence private, *images* of them.

**Def. 10.7**      **Theoretical perception** is the scientific explanation of empirical perception.

Theoretical perception describes a process of information transfer into the brain of the perceiver. For example, it explains vision by a process of electromagnetic radiation in the visible range being reflected off objects and then being focussed by the lenses of the eyes to form optical images of the objects on the retinas; these images are then transduced into neural images and transmitted down the optic nerves into the brain, whereupon the subject of that perception somehow becomes conscious of the pairs of them, united into single three-dimensional objects. The use of “somehow” here indicates a major lacuna in the theory.

**Def. 10.8**      **The general problem of perception** is the problem of deciding whether all that we empirically perceive around us is reality or images of reality.

We most of us have a strong conviction, almost immutable, in common sense realism: the belief that the empirical reality that we perceive around us is theoretically real — it continues to exist when unperceived, it exists independently of mind. But all the facts, scientific and philosophic, require us to perceive only images of reality, never reality itself, and these images are not theoretically real because their existence is perception dependent — just as the existence of a particular television picture is dependent upon the set being in working order and

receiving the signal. All the particular problems of perception are special cases of this general problem; twenty five of them are given here, and their relation to the general problem shown; eight earlier attempts at resolution of the general problem are then listed, and shown not to work; four arguments are then given for us perceiving reality, and three for us perceiving images of reality. Although this prolixity may be overkill, it is probably necessary in order to counteract the almost immutable belief in realism.

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1. The famous argument from illusion remains a problem because although its conclusion has frequently been renounced, the argument has never been shown to be invalid. This is the argument that because there is no intrinsic difference between illusions and non-illusions, both must be made of the same stuff; illusions must be images, or representations, of reality, since reality itself cannot be illusory, so non-illusions must also be images. Reality cannot be illusory because reality is our standard of truth and illusions are (dissimilarity) false ( Def. 4.6) perceptions, so illusions must be images. We cannot deny the fact of illusions, or their illusoriness. Illusions are contradictions, either between different senses, as between sight and touch in Aristotle's illusion and in the case of the half-immersed stick, or else between perception and well established belief, as in the case of visible objects appearing smaller with distance. So all that we perceive must be images of reality, of varying degrees of similarity to the real, not reality itself.

2. If you are looking at this book and you cross your eyes, or press on one eyeball, so that you have double vision, do you then see two real books, or two images of one book? If you see reality, then which is the real book: the book on the left or the book on the right? If each is an image rather than the real book, then as they coalesce when you uncross your eyes, do you still see two images, now coincident, or

do you see a real book? We believe that we now see a real book, but nothing has happened to change the images into reality.

3. If you watch a car drive down a straight road, it seems to get smaller as it gets farther away. You can hold up your thumb, in the manner of a landscape painter, and the size of the car, relative to your thumb, diminishes. We believe that the car does not really get smaller, so we distinguish between the real, or theoretical, size of the car, which is constant, and the apparent, or empirical, size which diminishes. At what distance does the apparent size equal the real size? Or, how far away must an object be for you to see its real size? If we perceive an image of a car then this problem is that of knowing when the size of the image is equal to the size of the real car; but if we perceive the real car then the real car must get smaller with distance.

4. It has been argued that real size does not vary with distance and measured size does not vary with distance, so real size and measured size are one and the same. But to argue so is to commit the fallacy of undistributed middle, as in: all women are human and all men are human, therefore all men are women. And the problem remains: how far from your eyes do a yardstick or a metre rule have to be in order for you to see a real yard or a real metre?

5. Secondary qualities are qualities which are manufactured by our sense organs, such as sensations of colour, which are manufactured by the eyes, or brain, according to the frequency of electromagnetic radiation that arrives on the retinas. So secondary qualities such as colours exist only within the nervous system of the perceiver. But when we perceive them they are out there, outside our nervous systems, in the world. The grass is both green and outside our heads. So where do secondary qualities exist: in our brains, or out there in the world? If they are real they exist outside our heads, while if they are images<sup>23</sup>

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<sup>23</sup> Properly speaking secondary qualities are called *representations* rather than *images*. Colours, sounds, smells, tastes, and feels represent certain molecular properties rather than are images; the images in theoretical perception are structures of secondary qualities.

they exist inside our heads, in our brains. Other secondary qualities are the various sensations such as sounds, warmth, and taste: in fact, *all* sensations are secondary qualities.

6. There are real things which cannot be perceived, but which can be described, such as atoms. So if colours are real, why cannot they be described to a blind man? The standard answer is that colours, being secondary qualities, are manufactured by the sense organs, and so are private to the perceiver. This makes colours images, hence unreal.

7. We believe things to be real by the fact of their being in public space. The space around us is public to everyone in it, and things in it are public by virtue of being in it. Mountains, trees, and people are all in public space, hence public, hence real. But the fuzziness of things seen with unfocussed eyes, or the redness of things seen through red glass, or after-images, or the bentness of the half immersed stick, are all in public space but unreal. We say that they are unreal, and hence illusions, either because they are private, or else because they are empirical contradictions. How can private illusions be in public space and remain private? The problem exists only because of the belief in the reality of public things, including public space. If this public space, being empirical, is an image then everything in it is also an image and there is no problem — other than explaining the externality and the publicity, which is not difficult. Also, how can empirical contradictions be in public space and so be real? The half immersed stick is bent to the sight and straight to the touch, so that if it is one stick then it is both bent and not bent at once, and thus a contradiction; and being perceived, it is empirical. And it is a fundamental principle of both science and philosophy that no contradiction can be either real or true. But if we only perceive images then we have a visual image — a bent stick — contradicting a tactual image — a straight stick — and there is no problem.

8. A normal swimming pool is cool to an overheated diver, and warm to a cold diver. So is the pool really cool, or really warm? It cannot be both, but if each diver has her own temperature representation, there is no problem.

9. When you see a rainbow, the concentric circles of colour, of which the rainbow is a set of arcs, have their centre on an imaginary straight line between the Sun, which is behind you, and your eyes: this line, extended forwards, goes to the centre of the circles of the rainbow. So as you move around, the rainbow moves with you, because your eyes are moving relative to the landscape. Consequently many people looking at a public rainbow each see their own rainbow, in a different position in the sky, and do not see the rainbows of the other people. Yet all these people agree that they all see the rainbow, that the rainbow is public. So is there one public rainbow, or are there many private ones? If there is one public rainbow then it is real, while if there are many private ones then they are images.

10. If two people are looking at a south facing house, one from the southwest and one from the southeast, then the first sees the front and west side of the house, and the second sees the front and east side. We say that each sees a different *aspect* of the house, and that every viewpoint yields a different aspect. So if every observer of the house only sees one aspect of it, at any one time, how can anyone ever perceive the real house? Alternatively, if the real house is the totality of its aspects, how can anyone ever perceive the real house? But if each aspect is a different image of the real house, there is no problem.

11. If, by some strange mutation, you were born always to see green as other people see red, and red as they see green, then as you learnt to talk you would have called your red, green, and your green, red. Could you ever know that your colour perception was different from other people's? You could not, of course, since experience of colours is private. But this means that the colours, being private, are images. Since everything you see is coloured, it follows that everything you see is an image.

12. If you were asked to point to something that you perceive which is entirely free of illusion, could you do so? If you could not then everything you perceive must be illusory to some degree, and hence an image.

13. And if you could point to something that is wholly non-illusory, how would you know it to be so? We have no way of knowing, just by observing, whether something is free of illusion. We know illusion is present if there is an empirical contradiction, but absence of such contradiction does not prove absence of illusion — just as the fact of Pat being pregnant proves that Pat is a woman, but absence of pregnancy does not prove that Pat is a man. Indeed, discovery of non-illusion is the task of empirical science — no simple undertaking.

14. When you see the Moon, which is 250,000 miles away, does your consciousness extend out of your head, for a distance of 250,000 miles, to the Moon, or do you see an image of the Moon, brought to you by reflected sunlight? If you see an image of it then you do not see the real Moon, while if you see the real Moon then your consciousness somehow has to get out of your head to that distance. So do you see the real Moon, or not? The real Moon and the image cannot be one and the same, because the Moon is made of rock, and the image is not made of rock.

15. If you do not see the real Moon, then is anything that you see real?

16. As Bishop Berkeley asked, if a tree falls in the forest and there is no one around to hear it, does it make a sound? If sounds are acoustical vibrations in the air, then the falling tree does make a sound, while if sounds are images caused by these vibrations then they exist only in brains and the falling tree does not make a sound. If we hear the vibration then we hear a real sound, and if we hear the effect of it then we hear an image of it.

17. When you talk on the telephone with a friend, do you talk with your friend, or do you talk with a reproduction of your friend's voice? Do you talk with your real friend, or an acoustical image of her?

18. Part of the process of perception is a largely unconscious *interpretation* of what we perceive. This includes an automatic *correction* of illusions, such as correction of visible size for distance; *compounding* of data from different senses into single objects, as when

a fire which is seen, heard, smelled, and felt is perceived as one fire; and *addition of beliefs*, such as the beliefs that perceived solid objects have far sides and continue to exist between occasions of being perceived. The purpose of interpretation, and its evolutionary survival value, is to make our perceptions more true. The problem is: what is it that we interpret, reality or images of reality? If we interpret reality then we make reality more true, which does not make sense, since reality is our standard of truth. But if we interpret images then we do not perceive reality.

19. We know that lenses appear to enlarge or to diminish things, and we also know that they do not enlarge or diminish the things themselves, but only images of the things. Photographs are images of things, and a photographic enlarger enlarges these images by means of lenses. Lenses change the sizes of images of things, not the sizes of things themselves. So everything seen through a lens must be an image, not the thing itself. A bacterium seen through a microscope, or a moon of Jupiter seen through a telescope, is only an image of the bacterium or the moon, not the reality. But our eyes have lenses, so everything we see must be an image, not reality.

20. What is the difference between hearing the siren of an ambulance, and hearing the sound of the siren? They cannot be one and the same because if you hear a recording of a siren then you hear the sound of the siren but not the siren itself. But in that case do you ever hear a siren?

21. Theoretical reality is either/or, it is not a matter of degree: something is either real, or it is not — just as things are either unique, or not. A real object cannot have illusory parts, because reality, like existence and consistency, is a distributive property (Def. 4.8). So if a whole is real then so is every part of it. To negate such a distributive property of a part is to negate it of the whole: if a part is non-existent then the whole has to be non-existent also, and if a part is inconsistent then the whole has to be inconsistent also. But an object seen is generally illusory with regard to its size, its aspect, its shape, and its colour, so has unreal parts, and so must be theoretically unreal. So how

can observed objects be partly illusory and partly theoretically real? If they are images, partly false and partly true, there is no problem; but if they are real objects the problem is insoluble.

22. If the Sun were to explode we could not know of it until eight minutes later, because that is how long it takes for light to travel from the Sun to Earth. So for eight minutes we would see an unexploded Sun, while the real sun would be exploded. It follows that we do not see the real Sun, we only see an optical image of it. But everything we see must be later than reality, because of the time it takes light to travel from reality to our eyes, so nothing we see is real, it is all images.

23. The optics of the lens of the eye require that the image that forms on the retina is upside down relative to its original, the real object. It used to be supposed that there was a  $180^\circ$  twist in the optic nerve, in order to explain the indisputable fact that the objects we see are the right way up; but physiologists found no such twist. So the inversion is now occult: supposedly it occurs somewhere, somehow, in the brain. But if the right-way-up empirical objects that we see *are* the images, no inversion is required.

24. There seems to be an incompatibility between theoretical perception and empirical perception, in that in theoretical perception there is a duality of real object and image thereof, while in empirical perception there is no such duality. It is sometimes supposed that a memory of an empirical object is the image of theoretical perception, but this cannot be. In theoretical perception the object, being real, is wholly non-illusory while the image is partly illusory; in empirical perception the object is partly illusory; so the empirical object must be the theoretical image, in which case the memory is an image of the image. Can this incompatibility be resolved, and if it cannot then which is false: empirical perception, or theoretical perception?

25. Stereoscopic vision gives us an empirical sensation of three-dimensional space through an unconscious construction of it, out of two two-dimensional images, one per retina. This is easily confirmed with the experience of stereoscopic photographs and movies. But this three-

dimensional space must be inside our brains, if it is constructed there, while we empirically perceive it as outside our heads. How does it get out there?

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We next look at eight failed solutions of the general problem of perception: namely, do we perceive reality or images of reality?

1. The most commons way of dealing with this problem is to argue that the real object and the image of it are similar, and therefore they are one and the same, identical. We speak, for example, of seeing the game on television, on the grounds that the television picture and the game are so similar that they may be regarded as one. But this, although a very common form of reasoning, is an error: the **identity error**, which is the contravention of a simple logical principle:

### **The principle that qualitative difference entails quantitative difference.**

This principle is so often abused that it is worth taking a moment to prove it. Suppose that we have two names or descriptions, A and B, and we want to know if they are identical (Def. 1.19) or not. If we know that there is a qualitative difference between what they each refer to, then we know that there is some quality, Q, say, such that A is Q and B is not-Q, or *vice versa*. If A and B are identical — one thing — then one thing is at once Q and not-Q, which is impossible. So A and B must be two. So qualitative difference entails quantitative difference. If the highest mountain in Africa has ice on top and Mount Kenya does not have ice on top then it is impossible for Mount Kenya to be the highest mountain in Africa, and if the President is clean shaven and Jack Robinson has a beard, then it is impossible for Jack Robinson to be the President.

In the present case, the image and its original are somewhat dissimilar and therefore cannot be identical.

2. Some of these problems of perception are commonly dealt with by supposing that we *project* internal things out into the world. We project secondary qualities such as colours onto external objects, for example, and as a result perceive the colours to be on the surfaces of the objects, even though we suppose that the colours are really in our brains. The problem with this is: how do we project? We do not project as movies and slides are projected onto screens, we do not project mechanically, as stones are projected from a slingshot and bullets from a gun, and we do not project geometrically, as the shapes of our shadows are projected on to the ground. But these are the only mechanisms of projection that there are. Unless an appropriate mechanism can be described, this solution is useless.

A second difficulty with projection is Problem 7: how can we project private things into public space and have them remain private?

A third difficulty is that some illusions cannot possibly be projected onto real things: how could smallness be projected onto a distant real object and make it small?

3. It is sometimes said that we perceive real objects *by means of* images of them: the images are brought into our brains by the sense organs and the afferent nervous system, and by means of these images we perceive their originals, the real objects. This can only work as a solution if the *means* can be spelled out. It is true to say that we can calculate quickly by means of computers, or boil water by means of electricity, or fly by means of airplanes; but it is false to say that we can levitate by means of will power, or speak with the dead by means of the Ouija board. This is because the first three means are genuine and the last two means are fictitious. So unless the means by which we perceive reality via images can be shown to be genuine, by being spelled out in detail, it is fictitious by default, in which case this explanation does not work. And no one has ever spelled out the details.

4. It is also sometimes said that we perceive reality *through* images, and so perceive both, as if the image was like a dirty window so that in looking through it we saw the scene outside as well as the dirt on the window. This does not work because the analogy fails. Images

are not like windows; nor are the retinas of our eyes like windows. The only window-like feature in vision are the transparent parts of the eyes, such as the lenses, and we do not see through these, we only see the images that they deliver to the retinas, or else neural transductions of these images.

5. Psychologists sometimes say that we unconsciously *organise* sense data in our brains, into two kinds: external data and internal data. Because of this organisation some of what we perceive is external and some internal: material and mental, as we usually call it. This obscures the fact that such organisation cannot put material data outside the perceiver's head if there is no doorway for it to go through. If you were in a prison cell with no egress, you could shuffle or arrange a deck of cards as much as you liked, but could not thereby organise any of the cards outside the prison: if you could so organise the cards, you could organise yourself outside as well.

6. The doctrine of *indirect perception* is the doctrine that to perceive something directly is to perceive its cause indirectly. To perceive something directly is to be conscious of it in empirical perception. So if we perceive the image of a real object, this real object is the cause of the image and so we perceive the real object indirectly, in which case we perceive the real object. This doctrine is seen to fail as soon as it is pointed out that *indirect perception* is a misleading way of talking about belief. Beliefs, like memories, are perception substitutes: when our perception fails us we substitute a belief. We cannot perceive beyond the horizon of the moment, but we believe that the world beyond it exists; and we cannot normally perceive the far side of opaque objects, but believe such objects to have far sides. So if we perceive images of real objects rather than the real objects themselves, then our perception fails us with the real objects: so we can only believe in the real objects, not perceive them. For example, lightning and thunder are caused, we believe, by atmospheric electric discharges; we cannot perceive these discharges, we can only perceive their effects, lightning and thunder. But can we be certain that they are caused by atmospheric electric discharges? The ancients believed that they were

caused by angry gods throwing thunderbolts, and if the ancients could be wrong, so could we. Beliefs may be false, but perception of reality cannot — since reality is the standard of truth. Thus the mere possibility of being false proves that *indirect perception* is belief, not perception, since perception of reality, if it occurred, could not be false. Furthermore, we can ask how far indirect perception might extend: if you read a newspaper, do you indirectly perceive the printing press, the editor, the journalist, the dramatic events that led to the story, the causes of these events, and so on? Or do you just believe in these things? If we could perceive indirectly to ultimate causes then we could know, rather than believe, whether or not there is a creator of the universe.

7. It has been suggested that we can perceive *through* causal chains, somewhat as we can perceive through a telescope. Thus transitively through a neural image, a retinal image, electromagnetic radiation, and molecular excitation, we see a real object. But why the real object? Why not the rods and cones on our retinas, or the electromagnetic radiation, or the Sun that illuminates the real object, or the cause of the real object, or the cause of this cause, or God? Quite apart from this, the analogy is patently wrong: there is as much similarity between a causal chain and a telescope as there is between a logical argument and a hearing aid.

8. Austin's famous distinction between *seeing as* and *it looks to me as*, which led to the ludicrous controversy as to whether illusions are adverbial or adjectival, is misleading. If I see a half-immersed stick and say "What I see is a straight stick but I see it as bent" or "What I see is a straight stick but it looks bent to me" then these statements do suggest that illusions are linguistic. But in fact both statements are false. The true statement is: "What I see is a bent stick but I *believe* that it is really straight." The illusion comes before its correction; the illusion is a matter of fact and the correction a matter of belief.

\* \* \*

We now consider four arguments that all that we perceive is reality, not images of reality, followed by three arguments to the contrary.

1. According to theoretical perception, real objects outside my head cause images inside my head. Since all that I empirically perceive is outside my head, it must all consist of real objects, not images of real objects.

2. Unless everyone is an extraordinarily consistent liar, what I empirically perceive is mostly public, while images inside my head are private. The public is perceived by other people, in public space, when it is not perceived by me, so exists independently of my consciousness. So all that I perceive which is public must be real, not images. Someone else may perceive my house, for example, while I am away, which proves that it exists independently of my perception, and so is real.

3. Most of what I empirically perceive is re-perceptible: when I return my perception to it, I perceive it again. The simplest explanation of re-perceptibility is that things are re-perceptible because they continue to exist between the times that they are perceived; this means that they exist while unperceived, and so are real, by definition. Particularly convincing cases of re-perceptibility are processes, such as a cake baking in the oven, logs burning in the fireplace, hands rotating around the dial of a clock, or the aging face of a friend. All of these can, and usually do, continue while unperceived, so that we perceive results rather than processes — which means that the processes are real, since they exist unperceived.

4. Images in our heads are mental but what we empirically perceive is material, in that it resists our wills while following scientific laws. If you step off a roof while willing yourself to levitate, Newton's laws will prevail over your will, thereby showing your body to be material and real, rather than mental and an image of reality. Another way of putting this is to say that the material behaves in a causally coherent manner, while the mental — dreams, for example, or political or religious belief systems — is usually incoherent. Causal coherence in general means that material objects conform to scientific laws. So what

we perceive is real because coherent, unlike our less coherent mental experiences.

Thus we may argue that all that we empirically perceive is real because it is external, public, re-perceptible, and material.

\* \* \*

Against this are three arguments that what we empirically perceive is all mental images, inside our heads.

1. Theoretical perception requires that we perceive images in our heads. This is shown by the fact that everything that we perceive is somewhat illusory, and theoretical perception explains illusions by making them distorted images of reality, in our brains. If theoretical perception, as a theory, is false, then so is much of theoretical science: electromagnetic radiation, acoustics, heat transfer, the chemistry of tastes and smells, etc. Only a scientific ignoramus can deny this much science, so for the rest of us what we perceive must be images of reality, not reality itself.

2. The second argument arises from the principle that qualitative difference entails quantitative difference.

We define:

**Def. 10.9**      The **empirical world**, of any one person, at any one time, is all that they empirically perceive at that time.

**Def. 10.10**    The **theoretical world** is all that is theoretically real.

Because of viewpoint and perceptual idiosyncrasies, everyone perceives an empirical world that is qualitatively different from everyone else's. Consequently there must be as many empirical worlds as there are observers. And if there are as many empirical worlds as there are observers, then because each of these worlds contains some illusion, each is qualitatively different from reality, or the theoretical world, which cannot contain any illusion, so none of them is the

theoretical world. The only way to make sense of this is to argue that each empirical world is an image of a portion of the real world, differing from other empirical worlds in its viewpoint. To suppose, as common sense does, that all of these empirical worlds and the theoretical world are identical, is to commit the identity error.

3. The third argument is simply that all the above twenty five problems with perception require that what we empirically perceive must be images of reality, not reality itself.

## 11. The Leibniz-Russell Theory

The above four arguments that we perceive reality, and the three that we perceive images of reality, all apply to everything that we empirically perceive — since we are still excluding introspection from empirical perception. Thus either everything we perceive is reality, or else everything we perceive is images of reality. But some of what we perceive is illusory, hence unreal, hence an image of reality. Therefore everything we perceive must be images of reality. Consequently we have to explain how images, which are internal to our heads, private, and mental, come to be external to our heads, public, re-perceptible, and material.

The key to such an explanation is what I have called the Leibniz-Russell theory. Leibniz' theory of perception differed from Russell's, which differs from what is offered here, except that all three have the key point in common. In his theory Leibniz<sup>24</sup> was trying to exclude all relations, while in his early theory Russell<sup>25</sup> was trying to maintain realism, by means of a six-dimensional space. But all three make the one point: if all that we empirically perceive is images of theoretical reality, then our own empirical bodies must be images — images of our theoretically real bodies. In particular, our own empirical

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<sup>24</sup>See Leibniz' *Monadology*. An outline of the logic of Leibniz' system is given in my *Renascent Rationalism*, obtainable from [www.speedside.ca](http://www.speedside.ca).

<sup>25</sup>Russell's early theory is given in two essays, *The Ultimate Constituents of Matter*, and *The Relation of Sense-data to Physics*, both of which are included in his *Mysticism and Logic*, 1918, and Penguin Books 1953. In this theory he held that at every point of theoretical space there is a three-dimensional empirical space. In his later theory, given in *Human Knowledge*, Allen and Unwin, 1948, he locates each empirical space in the theoretical space occupied by a theoretical brain. Russell used the terms *physical* and *perceptual* for my *theoretical* and *empirical*.

heads are images of our theoretical heads. Since everything that we empirically perceive is images inside our theoretical heads, this means that if you go outside on a sunny day then beyond the empirical blue sky is the inside surface of your theoretical skull.

With this understood, we may explain why the empirical is external, public, re-perceptible, and material, while at the same time it is internal, private, and mental.

1. **Externality.** When we theoretically perceive a theoretical object, that object is external to our theoretical heads and all three are imaged into our theoretical brains: the object, the relation of externality, and the head, are imaged as empirical object, empirically external to the empirical head. Empirical objects are external to the empirical head and internal to the theoretical head.

2. **Publicity.** If several theoretical people are in one theoretical vicinity then each theoretically perceives that vicinity: each experiences an image of it, as an empirical world, containing their own empirical body at the origin of their own co-ordinate system. These empirical worlds are largely similar, because they are images of one theoretical vicinity; they are somewhat dissimilar, because of distortions due to viewpoint and perceptual idiosyncrasies; and each is private to its perceiver, because no one can experience another's sensations. All this shows that there are two kinds of publicity and two kinds of privacy.

**Def. 11.1**      The observations of two people are **public by identity** if they both observe one and the same, or identical, thing.

**Def. 11.2**      The observations of two people are **public by similarity** if they both observe numerically distinct things which have a fair degree of similarity (Def. 5.26) between them.

A real object is public by identity to the theoretical perception of two people in one real vicinity. Their empirical images of this real object are public by similarity.

We have many everyday examples of publicity by similarity, or near similarity. All the contents of all the copies of one edition of a newspaper are public by similarity, as are all the television pictures of one program, on millions of different television sets, and all the renditions of one piece of music, on millions of record playings.

**Def. 11.3**      The observations of two people are **private by plurality** if they are not public by identity — if the observations are two, not one.

**Def. 11.4**      The observations of two people are **private by dissimilarity** if they have some degree of dissimilarity (Def. 5.27).

There are three consequences of these definitions: (i) publicity by identity disallows privacy by dissimilarity, since one thing cannot be dissimilar to itself; (ii) privacy by dissimilarity entails privacy by plurality, by the principle that qualitative difference entails quantitative difference; and (iii) publicity by identity and privacy by plurality are two-valued complements of each other: either one or the other obtains, never both; but publicity by similarity and privacy by dissimilarity are each a matter of degree, and are inversely related, and thereby are co-existent when both are less than unity.

Thus the several empirical worlds, each partly dissimilar and partly similar to the others, cannot be public by identity and so are somewhat public by similarity; and they must all be somewhat private by dissimilarity; and thereby must be private by plurality. They are thus both public and private.

3. **Reperceptibility.** Empirical objects are reperceptible not because they are real, but because they are images of theoretical objects. The empirical objects do not continue to exist between occasions of being perceived, but the theoretical objects do so continue, being theoretically real. Thus when theoretical perception is returned to a theoretical object a new image of it appears in the theoretical brain of the observer, who thereby reperceives an empirical object. The earlier

and later empirical objects are two, they cannot be one; to infer identity from their similarity is fallacious, the identity error.

4. **Material and mental.** This classification of the contents of consciousness is ultimately unworkable because of the fuzzy boundaries of the extensions of the concepts, but broadly speaking the material is external and public, while the mental is private. So empirical objects are material, in being external to the empirical head and public by similarity, and they are also mental in being internal to the theoretical head and private by plurality. And the behaviour of empirical things is causally coherent because theoretical reality is causally coherent and the empirical world images much of this feature of theoretical reality.

One way to emphasise all this is to point out that no two empirical worlds ever intersect, just as no two empirical minds ever intersect — because no two theoretical brains, and no two theoretical minds, can ever intersect; thus although they are often similar, empirical worlds are always disjoint.

Finally, one perhaps disturbing feature of the Leibniz-Russell theory is a consequence of the fact that each person's world may contain empirical other people: namely, as images, these empirical people are as mindless as photographs or television pictures of people: only their originals, the theoretical people, have minds.

## 12. Application of the Leibniz-Russell Theory

The twenty five problems of perception given in Chapter 10 all have obvious solutions, given the Leibniz-Russell theory.

The sixteen problems in philosophy of science given in Chapter 9 are also all soluble within the Leibniz-Russell theory, as follows:

1. The reason why there are two kinds of science, empirical and theoretical, is much more fundamental than the simple fact that some things are observable and others not; it is that there are two kinds of world: empirical and theoretical. Empirical science is the discipline of trying to describe the true, or non-illusory, features of empirical worlds, and theoretical science is the attempt to describe the theoretical world.

2. The basic relationship between empirical science and theoretical science is that empirical science describes empirical reality — everything empirical that is potentially universally public — and theoretical science explains empirical reality by describing theoretical reality — all that exists independently of mind. The theoretical world is the cause of empirical worlds, as is described by theoretical perception, and to describe causes is to explain their effects. Empirical entities are empirically perceptible, by definition, and theoretical entities are non-empirical, also by definition. The correctness of these definitions is clear from the fact that the theoretical world cannot be empirically perceived.

3. The relationship between theoretical reality and empirical reality is not identity, as common sense realism supposes, but a causal relationship, as well as a degree of similarity, or similarity truth.

4. The relationship between laws as generalisations and laws as theorems deduced within theories is not identity, as most scientists suppose, but degree of similarity, or similarity truth.

5. Concerning the probability of laws and theories, we remark first of all that the word *probability* has three meanings. One is mathematical probability, which is theoretical; the second is statistical

probability, or relative frequency, which is empirical; and the third is strength of belief, which is subjectively evaluated. Neither laws nor theories possess mathematical probabilities or relative frequencies, although they may state them, hence their probabilities must be the strength of scientists' beliefs in them. This is not as subjective as it seems initially, because there is generally consensus among scientists, which makes the probabilities public. The significance of publicity is explained in paragraph 8 below.

6. The reason why science is so much more successful than any other human means of gaining knowledge is that all the criteria of good empirical science and good theoretical science are criteria for truth: truth of observation, and truth of explanation. This will be more clear as these criteria are explained below, in paragraphs 8 and 16.

7. The nature of objectivity, so important in science, is that objectivity is attention to the public. This is obvious once it is understood that subjectivity is attention to the private, as when beliefs are determined by wish-fulfilment, prejudice, or vanity. The significance of publicity is explained in paragraph 8 below. Another answer to the problem of objectivity is that objectivity is rational (Def. 13.4), while subjectivity is irrational (Def. 13.5); this is equivalent to the first answer, since as we will discover in Part Four, the rational is much more public than the irrational.

8. The criteria of empirical science work because they all stress the public: objectivity is attention to the public, quantitative data are more public than qualitative data, and repeatability of experiments ensures that experimental results are public — by similarity. As we saw, empirical reality is defined as all that is potentially universally public, because such publicity is a necessary condition for non-illusion, which is similarity truth, or high degree of similarity, between empirical worlds and their originals, portions of the theoretical world. Thus empirical reality has the extrinsic property of similarity truth, which is established through similarities between, or publicity of, empirical worlds. We note that Einstein's principle of relativity is the principle that all the basic laws of science must be public to all

observers, regardless of their relative motion; and that the most potentially universally public content of minds is intensional mathematics.

9. The problem of induction is part of a larger problem, obvious once it is realised that scientific laws are not empirical but theoretical. This problem is that of knowing how we can have any theoretical knowledge at all, given that the theoretical world is empirically unknowable. This problem is dealt with in paragraph 13 below.

10. As to why the entities of theoretical science are all abstract entities, the simple answer is that the most successful branches of science are the mathematical branches, and mathematics is abstract. A better answer comes from the ontological argument, which requires the theoretical world to be identical with the best of all possible intensional mathematical worlds, and thereby consist of relations, which are abstract entities.

11. As to whether the abstract entities of theoretical science exist or not, the answer is that they do exist, according to the ontological argument.

12. As to where they exist, they exist beyond every horizon of the moment, according to the Leibniz-Russell theory: beyond the empirical blue sky of a sunny day and the empirical star-spangled black sky of the night.

13. Since our knowledge of theoretical entities cannot be empirical it must be speculative. All causal explanations are speculative, in that they attempt to explain empirical facts by describing their underlying causes. This applies to myth, metaphysics, theology, and the explanations of common sense, as well as to theoretical science. Underlying causes are causes that cannot be observed: they are non-empirical, theoretical, which means that they are in the theoretical world (if they do in fact exist) and so radically empirically imperceptible. Theoretical science is much more successful than other kinds of explanation, in explaining empirical data, because it is strictly disciplined by rules which have been found to work, over many generations of trial and error. These rules are the criteria of good

explanation, which are the criteria of theoretical science, discussed in 16 below.

14. The most important problem in all of philosophy of science is that of explaining how theoretical science is able to predict empirical novelty, both successfully and often. The explanation depends on relations of necessity (Def. 1.14): logical necessity in the theory and causal necessity in the theoretical world. The logical necessities allow the novelties to be deduced, and the causal necessities make the predictions of them come true. This explanation is impossible without the distinction of theoretical world and empirical worlds, since empirical worlds do not contain any relations of necessity, they only contain correlations. All of this will be much more clear with a concrete example, illustrated in Fig. 12.1, which uses pattern diagrams. We first need some definitions.

**Def. 12.1**      **Theoretical causation** is relations of necessity in the theoretical world. The antecedents of such relations are **theoretical causes** and the consequents are **theoretical effects**.

**Def. 12.2**      **Empirical causation** is exclusively extensional functions (Def. 3.10) among empirical data, characterised by universality. The arguments of these are **empirical causes** and their values are **empirical effects**.

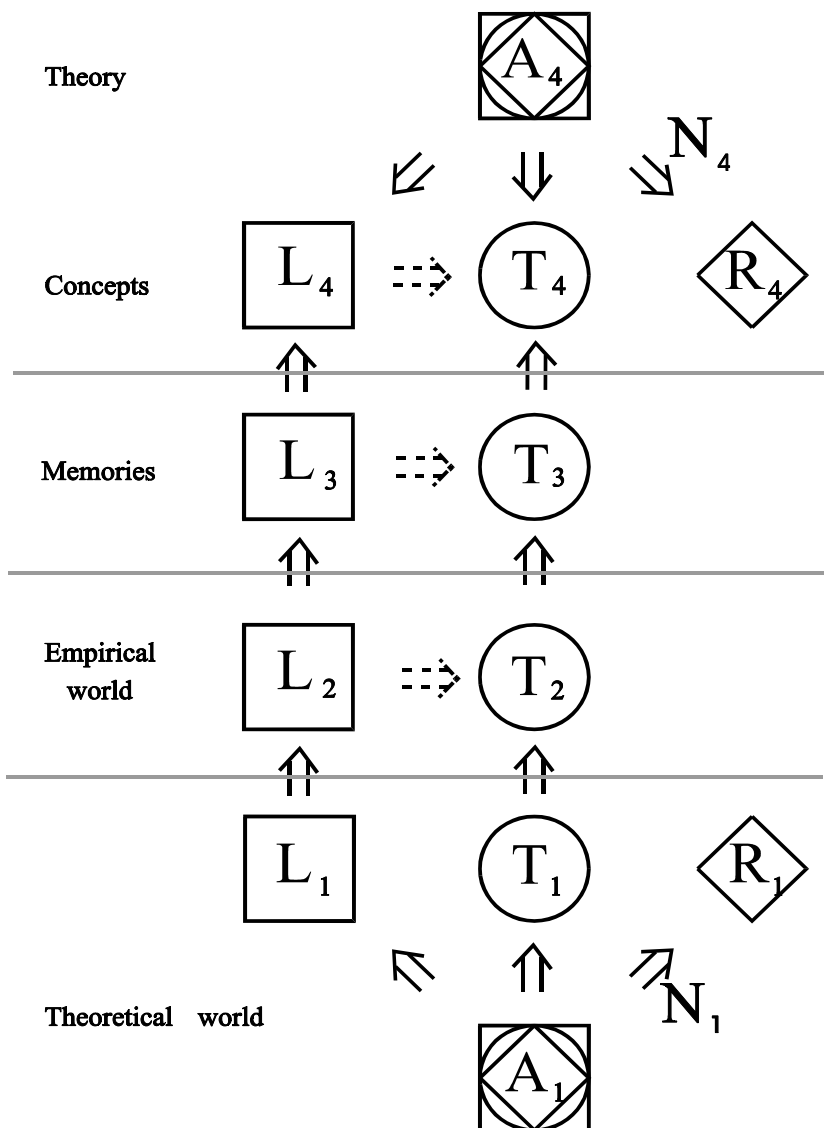


Fig. 12.1

Our example begins with a theoretical cause,  $A_1$ , and its theoretical effects,  $L_1$  and  $T_1$ . We may suppose, for clarity, that  $A_1$  is a

theoretical atmospheric electric discharge,  $L_1$  is a theoretical flash of lightning, and  $T_1$  is a theoretical clap of thunder. That is,  $L_1$  is a burst of electromagnetic radiation in the visible spectrum and  $T_1$  is a burst of atmospheric acoustical vibration, both of these radiating outwards in all directions as waves. Each of the theoretical effects theoretically causes, through the process of theoretical perception, a theoretical image of itself in an empirical world, inside a theoretical skull; these images —  $L_2$  and  $T_2$  — are empirically observed by the subject of that empirical world, as empirical lightning and empirical thunder, in the subject's empirica world. Each such empirical phenomenon theoretically causes an empirical image — a memory — of itself in that subject's empirical mind. Each such empirical image — remembered lightning and remembered thunder,  $L_3$  and  $T_3$  — then theoretically causes the formation of concepts —  $L_4$  and  $T_4$  — by a process that for now we simply take for granted, where a concept is a combination of a word or symbol and the abstract idea (Defs. 1.4, 13.32) that is its meaning. So  $L_4$  and  $T_4$  are concepts of lightning and thunder.

The heavy arrows in Fig. 12.1 represent the relation of necessity. Thus  $A_1$  theoretically causally necessitates  $L_1$  and  $T_1$ ;  $L_1$  similarly necessitates  $L_2$ , which necessitates  $L_3$  which necessitates  $L_4$ ; and  $T_1$  similarly theoretically causally necessitates  $T_2$ ,  $T_3$ , and  $T_4$  successively. (It is of course assumed that all other necessary conditions are present; for example,  $L_1$  would not necessitate  $L_2$  if there was no person present to theoretically perceive, and  $L_3$  would not necessitate  $L_4$  if the person concerned did not think about what he or she empirically perceives.) The broken empirical arrow between  $L_2$  and  $T_2$  simply — constant conjunction of contiguous and successive events, as David Hume put it, and in an empirical mind it is association of ideas. Thus the correlation between  $L_2$  and  $T_2$  is an instance of the scientific law that lightning empirically causes thunder; and a remembering or thought of lightning,  $L_3$  or  $L_4$ , is followed by a remembering or thought of thunder,  $T_3$  or  $T_4$ , by association.

If the person in whose mind all this is occurring has some basic

physics, he or she will have a concept,  $A_4$ , of an atmospheric electric discharge, and of how this causes thunder and lightning. The heavy arrow between  $A_4$  and  $L_4$  represents the relation of necessity within the theory that contains  $A_4$  and  $L_4$ , such that within this theory  $A_4$  necessitates  $L_4$  logically. In other words, the arrows from  $A_4$  represent logical necessity, as opposed to the causal necessity of all the other heavy arrows lower in the diagram. Similarly  $A_4$  logically necessitates  $T_4$ . Thus  $L_4$  and  $T_4$  can be deduced from  $A_4$ , and so can their correlation; hence the law that lightning causes thunder is explained in the scientists' sense of being deducible from the theory.

If the theory in the empirical mind of Fig. 12.1 is true, it is because it copies, as accurately as possible, a state of affairs in the theoretical world:  $A_4$  accurately copies  $A_1$ ,  $L_4$  accurately copies  $L_1$ ,  $T_4$  accurately copies  $T_1$ , and the logical necessities between  $A_4$  and  $L_4$  and between  $A_4$  and  $T_4$ , accurately copy the causal necessities between  $A_1$  and  $L_1$  and between  $A_1$  and  $T_1$ . Thus the law that lightning causes thunder is explained in the philosophers's sense of describing its causes.

It is such logical necessities that enable theoretical predictions of empirical novelty to be made, and such causal necessities that make them come true. Suppose now that  $A_1$  regularly causes  $R_1$ , as shown by the arrow  $N_1$ , but that  $R_1$  is not perceived as  $R_2$  because  $R_1$  is outside the range of human perception, so is theoretically imperceptible. In the present example,  $R_1$  is electromagnetic radiation outside the visible spectrum<sup>26</sup>. If the theory in which  $A_4$  necessitates  $L_4$  and  $T_4$  is true, then  $A_4$  will logically, mathematically, necessitate  $R_4$ , as shown by the arrow  $N_4$ . When the scientist deduces  $R_4$  from the theory, he is predicting novelty. In this particular case he would be predicting radio noise. An experimentalist should then be able to figure out what is needed — some experimental apparatus — to make  $R_1$  theoretically perceptible.

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<sup>26</sup>The visible spectrum is theoretically visible, but not empirically visible; the rest of the electromagnetic spectrum is not visible in either way.

When she has built this apparatus, a radio receiver, and set it up during an electric storm — a situation such that her empirical world will contain  $L_2$  and  $T_2$  — then, provided the radio works, she will hear radio noise,  $R_2$ , correlated with every flash of lightning,  $L_2$ , by means of it. The prediction of novelty will then have come true. It comes true because of the causal necessity,  $N_1$ .

(Note that the experimentalist builds two radios: her theoretical body, as willed by her ego (Def. 13.42), builds a theoretical radio; this is all theoretically perceived by her, so that her empirical body builds an empirical radio.)

I do not know whether any scientist ever made this particular prediction of novelty or not, but it could easily have been made and it serves well as an illustration even if it should be historically inaccurate.

Thus the possibility of prediction of novel empirical facts by theoretical deductions is explained. Minimum conditions for this are a four-way similarity between (i) the structure of the theoretical world, (ii) the structure of the public features of empirical worlds, (iii) scientific laws, and (iv) theories that are both constructs out of (iii) and reconstructs of (i). Without the theoretical world, theoretical prediction of empirical novelty is inexplicable, because there are no necessity relations in empirical causation to correspond to the logical necessity in the theory. In other words, if prediction of novelty is to be explained then believing in the existence of a world described by theoretical science is necessary; and, in particular, a world containing causal necessities described by the mathematical necessities in theoretical science. Without the mathematical necessities, such as  $N_4$ , the prediction of novelty would be mere guesswork, and without the causal necessities, such as  $N_1$ , the success of the prediction would be pure chance.

Thus the empirical fact of frequent, successful, theoretical, predictions of empirical novelty is good evidence for the truth of the Leibniz-Russell theory.

It is worth reiterating at this point that four pairs of L and T are necessary because of the facts of perception.  $L_2$  and  $T_2$  are bad copies

of  $L_1$  and  $T_1$  — just as correlation is a bad copy of necessity — and  $L_3$  and  $T_3$  are bad copies of  $L_2$  and  $T_2$ . We require this because of the facts of illusion and of inaccurate memories, and we know of these because of contradictions within our experience.  $L_4$  and  $T_4$ , on the other hand, are better copies of  $L_1$  and  $T_1$  because they are reconstructions, out of the data that are  $L_2$  and  $T_2$ , and other empirical representations, so as to exclude contradictions.

15. A closely related problem to that of explaining theoretical prediction of empirical novelty is the problem of why only mathematical theories can do this. The answer is that the theoretical world is relational and so is described by intensional mathematics.

16. Finally, why do the criteria of theoretical science work? To recapitulate, the falsifying criteria are: (i) contradiction within a theory and (ii) theoretical contradiction of empirical data. The verifying criteria are (iii) the size of the scope of the theory, (iv) the density of detail within a given scope, (v) successful prediction of empirical novelty, (vi) integration with other accepted theories, (vii) simplicity of theory, (viii) beauty of theory, and (ix) symmetries within the theory.

The first four are self-explanatory. The falsifying criteria work because neither empirical reality nor theoretical reality may contain contradictions, and because theories must resemble empirical reality; and scope of explanation and density of detail within a given scope both require that the more empirical detail a theory explains, the better an explanation it is.

Successful prediction of novelty has already been explained; as we saw, its occurrence is a convincing reason for belief in the truth of the theory from which it emerges.

The remaining criteria work because of the nature of the theoretical world: it is an integrated whole, and it is simple, beautiful, and symmetrical. We know all of this from the ontological argument.

17. A precise statement of the method of science has long been a goal of philosophers of science, since if the method can be spelled out then it might be applicable to other fields of human investigation. The best known such attempt was Mill's *Methods*, consisting of five ways

of discovering correlations between empirical data. These methods are non-mathematical and non-theoretical, and do little more than show the difficulty of discovering scientific method. This difficulty exists because science is creative and we have no method for creativity. When creativity is abundant we call it genius, and the need for genius in science is shown by the fact that the invention of great theories and the design of great experiments both require genius (Def. 16.2): genius which is innate and so cannot be taught. The average scientific researcher, although more talented scientifically than the average human being, is incapable of creating theories like Newton, Maxwell, or Einstein, or designing experiments like Faraday, Hertz, or Rutherford.

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**Def. 12.3**      **Intensional science** is science whose theory is applied intensional mathematics.

**Def. 12.4**      **Exclusively nominal science** is all false claims to empirical data, scientific laws, or theories.

**Def. 12.5**      **Exclusively extensional science** is the science of empirical extensional sets; it is neither intensional nor exclusively nominal.

Theories that closely approximate the truth must be intensional, since, as we have seen, theoretical reality is both intensional and our standard of truth. But not all science has such theories. A notable example was that failed branch of psychology called behaviourism, which, through a misunderstanding of the nature of science, denied all theory. Other branches of science, such as medicine and the social sciences, lack good theory because of the complexity of their subject. Where there is little or no intensional theory, researchers have to fall back on an extensional approach to science. The mathematics of

contingent extensions is, of course, **statistics**. The three major practical differences between intensional and extensional scientific research are (i) intensional science deals with causes — intensional necessities — while extensional science deals with correlations, or extensional necessities; (ii) because of this, intensional science needs only enough empirical instances to establish publicity of experimental results, while extensional science needs a statistically significant number of instances; and (iii) the use of **control groups** is necessary in the extensional sciences, but not the intensional sciences.

It is sometimes possible to predict novelty in extensional science, by interpolation. One example of this was Harvey's prediction of the existence of the capillaries, before the invention of the microscope, in order to account for the circulation of the blood; he interpolated the capillaries from the gaps between the smallest visible arteries and the smallest visible veins. Another example was the many predictions of new chemical elements, interpolated from gaps in Mendelieff's periodic table.

Nominal science includes reported false experimental results, such as cold fusion, and theories which failed because their invented entities, such as phlogiston and the electromagnetic ether, had no real reference. Nominal science also includes theories, such as vitalism, creationism, and parapsychology, which are believed for subjective reasons but are false for various other reasons: they may be contrary to empirical data, inconsistent, lacking density of detail, inferior to their alternatives, or unable to integrate with other probable theories.

The separation of nominal science from intensional and extensional science is a matter of consensus among scientists, or publicity of belief.

## PART FOUR

### Intensional Philosophy of Mind

#### **13. *Mind.***

The main goal of our theory of mind is explanation of empirical mental facts, including the fact that a mind can think mathematically. Another goal is to explain the possibility of knowledge of the best of all possible intensional mathematical systems. Four particular things in all of the foregoing enable such a theory of mind: namely, the nature, and reality, of relations; the evolutionary need to increase hekergergy; the significance of fields in explaining hekergetic causation; and the Leibniz-Russell theory of perception. And since the quality of an explanation depends upon the density of detail that it explains, we demonstrate the adequacy of the present theory with considerable detail, even though not all of it is mathematical.

**Def. 13.1**      An **atomic idea** is a neural switching, an on or an off, in a theoretical brain.

For the sake of clarity, these will be capitalised: On and Off. They exist in a theoretical space and time within the theoretical brain, and are the basis of a theoretical mind.

From the evolutionary need to increase hekergergy we derive:

**The mind hekergergy principle:** a mind changes so as to increase hekergergy, or, if this is not possible, so as to maintain it, or, if this is not possible, so as to minimise its loss.

**Def. 13.2** A **need** is a state in a theoretical mind to which the mind hekergy principle applies.

Thus a need is a need to increase a particular hekergy, or maintain another, or minimise the loss of a third.

An On is like, or similar to, or resembles, another On and dissimilar to, or unlike, an Off, and an Off is similar to, or like, another Off. From this, and the mind hekergy principle, we derive:

**The Principle of L.A.L.R.U.:** structures of atomic ideas act on a basis of like-attracts-like-and-repels-unlike (abbreviated to L.A.L.R.U., or L.A.L. for short). The force of attraction or repulsion is assumed to be analogous to other forces, with the exception that it is conditioned by degrees of similarity. That is, it is proportional to the product of the hekergies of the structures, and to the degree of similarity between them, and inversely proportional to the square of the distance between them:

$$F \propto (2n - 1) \frac{H_1 H_2}{d^2} \quad (13.1)$$

where  $F$  is the force,  $n$  is the degree of similarity between the structures,  $H_1$  and  $H_2$  are their hekergies, and  $d$  is the distance between them.  $F$  is attractive if positive and repulsive if negative, and  $1 \geq n \geq 0$ .

We note that L.A.L. usually is a hekergy increasing process.

**Def. 13.3** A **theoretical mind** is all the atomic ideas in a theoretical brain, plus all innate ideas, plus all data that are brought in by the theoretical afferent nerves, plus all that emerges cascadingly out of these.

A theoretical mind probably consists initially of only Offs, like Locke's *tabula rasa*, or blank tablet, except for a few innate theoretical ideas needed to explain certain empirical facts, such as a new-born infant's recognition of a smile and its ability to suck. We may suppose that from this starting point structures of atomic ideas flow into the mind from the afferent nerves, in accordance with theoretical perception.

**Def. 13.4**      The **irrational** is anything in a theoretical mind that is structured by L.A.L.

**Def. 13.5**      The **rational** is anything in a theoretical mind that is structured with maximum hekerger.

**Def. 13.6**      An **agent** is anything which has some awareness (Def. 13.19) of, and some control (Def. 13.42) over, its environment.

As this theory develops, we will find that a theoretical mind includes three agents: an ego, an anti-ego called an oge, and a third agent called the psychohelios; the first two will be found to be irrational and the third rational.

Besides L.A.L., we suppose two basic processes within a theoretical mind: mapping and bonding.

**Def. 13.7**      A theoretical idea — a structure of atomic ideas — is **mapped** into another idea when a reproduction (or image, or copy) of it is produced as that other idea.

**Def. 13.8**      Two theoretical ideas are **bonded** when they are permanently joined together.

The material on which mappings and bondings work is, initially, theoretical sensations:

**Def. 13.9**      A **theoretical sensation**, or **mid-sensation**, is a level-two structure of atomic ideas, brought into the theoretical mind by theoretical perception.

Theoretical sensations exist only for as long as theoretical perception is producing them. They are representations of properties in the real world. They are called mid-sensations because they are causally mid-way between the theoretical world and the content of empirical perception; this latter is explained shortly. The qualifier *mid-* is also used for other structures in the theoretical mind which are mid-way between the theoretical and the empirical.

**Def. 13.10**      A **mid-object** is a level-three structure in a theoretical mind, a structure of theoretical sensations.

**Def. 13.11**      A **mid-world** is a complete structure of mid-objects.

Mid-objects are images of real, or theoretical, objects, which are wholes in the theoretical world; and mid-worlds are images of the vicinity of the theoretical world which contains the theoretical perceiver: everything within the range of the perceiver's theoretical perception. Mid-worlds are transient, succeeding each other for as long as theoretical perception continues.

**Def. 13.12**      A **mid-memory** is a relatively permanent image of a mid-world, or a part of a mid world, mapped from that mid-world.

**Def. 13.13**      The **mid-body** is the mid-object that is an image of the theoretical body of the person who is theoretically perceiving.

**Def. 13.14**      **Ego-memories** are mid-memories that contain a mid-memory of the mid-body.

**Def. 13.15**     The **ego** is a structure of ego-memories, mutually attracted by L.A.L. because of their common feature of a mid-memory of the mid-body.

A mid-sensation in the vicinity of the ego will distort, or stress, the ego through L.A.L. forces.

**Def. 13.16**     An **empirical sensation** is the L.A.L. reaction in the ego to a mid-sensation; it is a poor image<sup>27</sup> of a mid-sensation, mapped by L.A.L.

**Def. 13.17**     An **empirical object** is the L.A.L. reaction in the ego to a mid-object, hence an image of that mid-object.

A mid-object is a structure of mid-sensations, and so an empirical object is a structure of empirical sensations.

**Def. 13.18**     An **empirical world** is a structure of empirical objects, an image of a mid-world.

**Def. 13.19**     The **awareness**, or **consciousness**, of the ego is the presence of empirical objects within the ego; they are the objects of this consciousness.

Thus the ego is conscious, over time, of a series of transient empirical worlds. This consciousness is empirical perception. Any empirical world, empirically perceived at any one time by an ego, is thus entirely within the structure of the ego: beyond the empirical blue

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<sup>27</sup> All empirical sensations are so-called *secondary qualities*, mental products of neural stimuli, and concrete: colours, sounds, tastes, etc. As such they are sometimes called *representations* of their originals, rather than *images* of them; but we will generally call them images, for simplicity.

sky are the outer limits of the ego; and, transitively, beyond those outer limits is the inside surface of the ego's **theoretical skull**.

Every empirical world has an empirical body, which is the origin of a subjective co-ordinate system; once it has speech, the ego might describe this origin as "I, here, now," and the co-ordinates as "My left," "In front of me," "My future," etc. A mature ego will be able to transpose this co-ordinate system on to another, such as a geocentric one or a heliocentric one; and then recognise that scientific laws have to be independent of all co-ordinate systems.

Because a mid-object may be a complex structure of mid-sensations, and there is a fair variety of the latter, it follows that the L.A.L. reactions within the ego — itself a complex structure — are complex and so the consciousness of the ego is complex. However, complex as the ego's consciousness is, it is trivial compared with the complexity of the theoretical structures that give rise to it; the shape and colour of an empirical green leaf, for example, are simple compared with the cellular and molecular structure of the theoretical leaf — as the simplicity of the sensation of green and the complexity of chlorophyll molecules show.

The L.A.L. forces that produce the consciousness of the ego depend upon the structure of the ego, and the hekerger of the ideas within it, as well as upon the mid-objects that are the immediate causes of the consciousness — in accordance with the formula (**13.1**).

**Def. 13.20**     The **attitude** of the ego is the effect of its permanent structure on its consciousness.

The most characteristic feature of the structure of the ego is that it is composed of mid-memories of its own mid-body, which are the most basic ideas of the self. Later, the ego itself becomes the meaning of self. Once it has language, the ego might say "I fell head over heels" and "I dreamt that I was disembodied," thus illustrating the two meanings of self. The significance of the ego as self is that the ego, in

accordance with the mind hekerger principle, must try both to preserve and to increase its own hekerger.

**Def. 13.21**     **Selfishness** is the attitude of the ego that results from the mind hekerger principle.

We must suppose that the ego is capable of focussing its consciousness, on one or more among a group of mid-objects, so as to concentrate on those most likely to provide an increase of its hekerger.

**Def. 13.22**     **Attention** is the ego's focussing of its consciousness.

Attention, most basically, focusses on three kinds of content of empirical consciousness: sensations, relations, and hekergeries. Sensations include colours, sounds, touch, tastes, smells, and kinaesthetic sensations. Relations between sensations may be empirically perceived individually, or constitute structures of sensations, such as empirical objects. Relations between other relations, and properties of relations, may also be empirically perceived, and as such may be mathematical entities. Hekergeries of relations are **values**, as claimed in Chapter 7, and consciousness of a hekerger is consciousness of an empirical value.

**Def. 13.23**     **Absolute values** are theoretical values, actual hekergeries.

**Def. 13.24**     **Empirical values**, or **human values**, are the ego's consciousness of images of absolute values.

Empirical values are subjective because consciousness of images of absolute values is conditioned by the structure and hekerger of the ego, in accordance with the formula (**13.1**).

Consciousness is usually dynamic: attention does not remain static for long.

**Def. 13.25**     **Feelings** are the ego's dynamic attention to values, as opposed to **thoughts**, which are its dynamic attention to sensations and relations.

**Def. 13.26**     A **goal** of the ego is any possibility of its own hekerger increase of which the ego is conscious.

By definition, a goal is based on value; it represents the fulfilment of a need. Attention to a goal leads to action (Def. 13.42) by the ego.

**Def. 13.27**     **Pleasure** is the ego's consciousness of hekerger increase and **pain** is its consciousness of hekerger decrease.

Pleasures and pains are of two kinds: mental, which are hekerger increases or decreases of the ego, and material, or bodily, which are increases or decreases of the hekerger of the body. The former the ego experiences directly, the latter it experiences as images.

**Def. 13.28**     **Desire** and **aversion** result from the mind hekerger principle acting on the recognition (Def. 13.39) of the possibility of pleasure and pain.

**Def. 13.29**     An **empirical memory** is the ego's consciousness of a mid-memory.

**Def. 13.30**     A **concrete quality** is any empirical sensation.

**Def. 13.31**     A **concrete idea** is an empirical memory of a concrete quality, or a structure thereof.

A structure of concrete qualities is, of course, an empirical object. Concrete qualities are the elements of empirical perception, as opposed to atomic ideas — Ons and Offs — which are the elements of

theoretical minds, and separators, which are the elements of the theoretical world. Thus empirically a sensation is a level-one structure, an object is a level-two structure and a world is a level-three structure.

**Def. 13.32**     An **abstract idea** is any intensional meaning in the theoretical mind.

This definition is thus an amplification of our earlier Def. 1.4, of an abstract idea as any ideal relation. We could distinguish between theoretical (or mid-) and empirical abstract ideas, the latter being the ego's consciousness of the former, but little is gained thereby; so unless the context requires it, the distinction will be avoided, for the sake of simplicity. The same goes for subsequent definitions, such as theoretical and empirical propositions: the distinction will be made only if needed.

The ego may manipulate ideas at its periphery by appropriate focussing of its consciousness. Its consciousness of these processes is either imagination or thought.

**Def. 13.33**     **Imagination** is the ego's manipulation of, and consciousness of, concrete ideas.

**Def. 13.34**     **Thought** is the ego's manipulation of, and consciousness of, abstract ideas.

**Def. 13.35**     A **proposition** is a structure of abstract and/or concrete ideas.

**Def. 13.36**     A **belief**, by the ego, is a proposition that is incorporated into the structure of the ego, by L.A.L.

Thus the ego consists of mid-memories and mid-beliefs, and also of innate ideas. This is quite plausible, in that, once it has speech, any ego might say existentially "I am what I have inherited, experienced, and done, and what I believe." Notice the fundamental

difference between an ego considering a proposition, and believing it: in the first case the ego is simply conscious of the proposition, while in the second the ego incorporates the proposition into itself.

**Def. 13.37**     A **prejudice** is a structure consisting of a belief and supporting evidence for that belief; by L.A.L. the belief attracts evidence in favour of itself and repels evidence against itself.

A prejudice seems obviously true to its possessor because of the presence of all the supporting evidence, and the lack of contrary evidence; and the possessor is unaware of the L.A.L. process of selection of evidence, so is unaware of the prejudicial nature of the belief.

**Def. 13.38**     **Classification** is the process of collecting similar ideas into sets, by L.A.L.

Because the similarities between ideas are usually of degree less than one, the similarities are only fadingly transitive; hence many classifications are fuzzy (Def. 5.30).

**Def. 13.39**     **Recognition** results from the comparison of a present perception with a memory, such that the comparison yields similarity; the memory is attracted to the present perception by L.A.L. and the recognition is consciousness of the similarity.

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**Def. 13.40**     A **motor-idea** is a mid-idea that may be sent down the efferent nervous system so as to produce a specific movement of muscles in the theoretical body.

By analogy with computer theory, motor-ideas are instructions rather than data, yet consist of the same stuff: structures of atomic ideas.

Movement of theoretical muscles is, of course, imaged into movement of mid-muscles, by theoretical perception, and hence imaged into movement of empirical muscles, by empirical perception.

**Def. 13.41**     An **action-point** is the point in the theoretical mind, at which a motor-idea is delivered to a set of efferent nerves.

There are different action points for different muscles, or sets of muscles. We may suppose that the ego is capable of moving motor-ideas to any necessary action point, by a combination of L.A.L. and attention, in accordance with its selfish needs.

**Def. 13.42**     **Action** by the ego is control of the theoretical body by means of motor-ideas. The movement of motor-ideas to their action points by the ego is the **willing** of that action by the ego.

Because the results of willing by the ego appear in the consciousness of the ego as movements of the ego's empirical body, there is a feedback loop between a goal of the ego, the action to achieve that goal, and its achievement. This might be as simple as picking up some food and putting it in the mouth: the ego is conscious only of an innate desire, hunger; of empirical food; of empirical action; and of satisfaction; whereas the actual process involves the cause of the hunger in the theoretical body; mid-hunger; empirical hunger; action; movement of the theoretical body, hence of the mid-body and the empirical body; theoretical taste of the food, hence mid-taste and empirical taste; and theoretical satisfaction, hence mid-satisfaction and empirical satisfaction. Each of these triads of theoretical, mid-, and empirical is believed to be a unity by common sense, in its passion for simplicity.

A particular movement is remembered, and this memory is an idea of the movement. Willing, by the ego, presupposes that the ego is conscious of the movement it wants; so the mid-idea of the movement, and the motor-idea that produces it, must be bonded. Production of such bondings is the process of an infant ego learning control of its own empirical body.

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A special case of action by the ego is speech, and later writing, which presuppose language and the ego's comprehension of language.

**Def. 13.43** A **word** is a structure of theoretical and empirical ideas bonded together, consisting of a memory of a sound, the motor ideas to produce a similar sound, and a theoretical idea that is the meaning of the word; it usually also has bonded to it a memory of a written word and the motor ideas to produce a similar written word; and it may have similar structures bonded to it that are synonyms, and special symbols or foreign words having the same meaning.

Most words have as their meanings a class of similar ideas, formed by L.A.L., rather than a single idea. For example, the meaning of “My house” is a set of memories of my house, as well, perhaps, as present perceptions of it. An example of a single idea being a meaning is a memory of a particular event — although most memories of events are memories of processes rather than of single events.

**Def. 13.44** The **concrete meaning** of a word is extensional-any (Def. 2.35) one member of that class of concrete memories which is the meaning of the word.

**Def. 13.45** A **proper name** is a word bonded to a single object.

**Def. 13.46**     A **concrete name** is a word bonded to a concrete meaning.

**Def. 13.47**     A **concept** is a word bonded to an abstract idea.

A proper name is actually bonded to a class: a class of distinct memories of one theoretically perceived object; common sense, and ordinary language, usually confuse the similarities of these memories with identity. A concrete name is also bonded to a class, but a class of concrete memories of many, similar, theoretically perceived objects. A concept is also bonded to a class, but it differs from a concrete name in that it is bonded to an intensional meaning, whereas a concrete name is bonded to an exclusively extensional meaning; also, concrete names have concrete ideas as their meanings and concepts have abstract ideas as their meanings. Words, in this context, include symbols such as mathematical symbols.

Descartes and Spinoza distinguished between what they called universal notions and common notions. Universal notions, or inadequate ideas, are confused images which belong in the imagination, while common notions are adequate ideas, abstract ideas which belong in thought; clearly, the present concrete and abstract names are their universal and common notions.

**Def. 13.48**     **Grammar** consists of rules that relate words and the meanings of words.

**Def. 13.49**     A **sentence** is a set of grammatically related words.

The meaning of a sentence is usually<sup>28</sup> a proposition (Def. 13.35). Communication of propositions is possible between two or more theoretical people if they speak the same language. Person A may

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<sup>28</sup>Other meanings may be questions, exclamations, orders, etc. Propositions are distinguished from these by being either true or false.

will the speaking of a sentence by willing the motor ideas of the words to their action point; this causes the theoretical muscles to produce the sounds of the words in the theoretical world, where they may be theoretically perceived by person B, who is conscious of images of them as empirical sounds. Person B will thus empirically hear the sentence and, having similar words and grammar, will recognise (Def. 13.39) the words and so be conscious of the meaning of the sentence. A similar process occurs with writing.

Note that for person B, there is an empirical person A, in person B's empirical world, who appears to be the source of the speech; but of course this empirical person A has no mind, being merely an image of the theoretical person A, who does have a mind.

Given language, we may distinguish three kinds of thought:

- Def. 13.50**     **Pure thought** is manipulation of abstract ideas — intensional meanings — without associated language.
- Def. 13.51**     **Ordinary thought** is manipulation of abstract ideas with the aid of language — manipulation of concepts.
- Def. 13.52**     **Calculation** is algorithmic thought: manipulation of words without their associated abstract ideas, in accordance with rules derived from ordinary thought.

Pure thought occurs when a new abstract idea is created but has no name, or when the ego is attending to meanings rather than words, or when the ego is conscious of an abstract idea but the word for it is unavailable — it is “on the tip of the tongue.” Ordinary thought attends to both language and meaning at once. Calculation is sometimes described as “silent speech.” Nominalists are those who believe that all abstract thought is nothing but silent speech, and thereby has nominal meaning only.

Thought is rational, or largely so, in accordance with the mind hekergy principle — unlike everything else in this and the next Chapter, 14, which is irrational because determined by L.A.L.

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- Def. 13.53**     **Discrimination** is consciousness of the results of special mappings, such as the following:
- Def. 13.54**     A **patch mapping** is a mapping of a minimal area of a concrete original, just large enough to be within consciousness.
- Def. 13.55**     A **boundary mapping** is a mapping of boundaries — of contiguous dissimilarities.
- Def. 13.56**     A **scale mapping** is a mapping with uniform enlargement or diminution.

Discrimination depends upon *comparisons*, which were described as bipossibility relations each of whose two consequents are similarity or dissimilarity. They emerge in structures sufficiently complex to produce them, such as animal minds and computers. The present theory of mind may be supposed to be sufficiently complex to produce them.

Two patch mappings of two sensations may be compared, resulting in the emergence of a relation of similarity or dissimilarity. This relation will be vocalised as *same* or *different*, so that the ego might say “This colour is the same as that colour” or “This tastes differently from that.” The comparison of boundary mappings will give same or different shapes, and scale mappings will give relative sizes — of lengths, areas, or volumes. Other special mappings may be supposed for other kinds of discrimination, although some discriminations are simply comparisons.

A major discrimination is between the mental and the material:

**Def. 13.57**     The **material** is everything in the consciousness of the ego that is external, public, perceptible, and causally coherent.

**Def. 13.58**     The **mental** is everything in the consciousness of the ego that is not material.

The distinction between the material and the mental breaks down under close analysis, but is important to those egos who include the false belief in realism — the belief that material objects continue to exist when unperceived.

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**Def. 13.59**     **Vanity** is a false belief that the hekerger of the ego is greater than its actual value.

Vanity is produced by the ego's incapacity to increase its own hekerger, and the need to do so arising from the mind hekerger principle. It is consequently most prevalent in immature egos.

**Def. 13.60**     **Rationalisation** is the manufacture, by the ego, of a false belief that supports vanity.

Rationalisation usually occurs as a result of a failed attempt to increase hekerger.

Both vanity and rationalisation are examples of how L.A.L. — irrationality — can increase the falsity within a mind.

## 14. The Oge.

Among the irrational features of mind which prevent knowledge of the best of all intensional mathematical systems are internal conflict and social pressures. Both of these require the concept of oge for their explanation.

**Def. 14.1**      The **oge** (rhymes with *bogey*) is a second agent (Def. 13.6) in the theoretical mind which grows out of mid-memories of other people; as such it represents, and acts on behalf of, society.

**Def. 14.2**      **Approval** is evaluation of an action as hekerger increasing; **disapproval** is evaluation of it as hekerger decreasing.

Most ego-memories include memories of other people. These memories are both like ego-memories in that they contain a memory of the ego's body and actions, and also unlike in that they contain a memory of someone else. Someone else is unlike the ego's body because it cannot be controlled directly by ego action, and also because it is not located exactly at the origin of the subjective co-ordinate system. These compound ego-memories are therefore both attracted to and repelled from the ego, by L.A.L. These memories also include the attitudes of these other people towards the ego: approving or disapproving. An approving memory is attracted to the ego by L.A.L., and a disapproving one is repelled, by L.A.L. thus approval and disapproval determine whether the memory is, on balance, attracted to the ego or repelled from it.

The disapproving ones are all similar in their disapproval and so are mutually attracted by L.A.L., as well as repelled from the ego; the structure that they form by mutual attraction is the oge. The oge is an agent, conscious and able to act, for the same reasons that the ego is.

Like the ego, the oge is composed of beliefs as well as of memories. These are formed in the oge by introjection:

**Def. 14.3**      **Introjection** is the incorporation of a proposition into the oge, thereby making it an oge-belief, as a result of another person asserting this proposition in the hearing of the ego.

We saw that the basic attitude of the ego is selfish, this being an application of the mind hekerger principle to the self, which is the need to increase the good, or hekerger, of the self. In similar fashion the application of the mind hekerger principle to the basic attitude of the oge is the need to increase the good, or hekerger, of society.

**Def. 14.4**      The **moral** is any increase in the hekerger of society, and hence of the oge.

Clearly, the goals of the oge are moral — although some exceptions to this will be found later.

**Def. 14.5**      **Guilt, shame, embarrassment,** and **conscience** are feelings produced in the ego by the oge.

Guilt and shame result from the oge's reaction to immoral behaviour by the ego; guilt arises from private immorality and shame from public. Embarrassment results from the oge's reaction to the breaking of a taboo (Def. 14.7), such as the nudity taboo. Conscience results from the oge's reaction to immoral ego-memories.

**Def. 14.6**      **Conflict** occurs when there are two agents having mutually exclusive goals in a common situation.

Since ego goals are increases of the hekerger of the ego, and oge goals are hekerger increases of the oge, their goals are usually mutually exclusive; they also have a common situation, since the empirical world

of each is an image of one mid-world; hence they are usually in conflict. This conflict may be of two kinds: inclination-duty conflict, and neurotic conflict, as will be explained in greater detail shortly.

**Def. 14.7** An **oge-inhibition** is the prevention, by the oge, of an action willed by the ego; an **ego-inhibition** is the prevention, by the ego, of an action willed by the oge. A **taboo** is a very strong oge-inhibition.

Oge-inhibitions are such things as shyness and inarticulateness; examples of ego-inhibitions are disobedience and truculence.

**Def. 14.8** An **oge-compulsion** is the forcing, by the oge, of an action against the will of the ego; an **ego-compulsion** is the forcing, by the ego, of an action against the will of the oge.

Oge-compulsions are such things as blushing and self-sacrifice; examples of ego-compulsions are taboo-breaking, defiance, and profanity.

**Def. 14.9** An **oge-person** is a set of mid-memories of one person, in the oge.

The shape of the oge is different from the shape of the ego because the ego consists of mid-memories of one person and the oge of many different people. The shape of the ego is more or less spherical. The shape of the oge is initially linear, and polarised, with the most approving oge-persons at one end and the most disapproving at the other. These ends are characterised by good and evil, and love and hate.

**Def. 14.10** **Good** is hekerger increase.

**Def. 14.11** **Evil** is hekerger decrease.

Although evil may be of either natural or human origin, the oge is primarily concerned with the latter. Natural evils include hurricanes, earthquakes, volcanoes, famines, disease, etc. Human evils include war, crime, and malice.

**Def. 14.12**     **Love** is a willingness to raise the hekerger of another person, unconditionally. (See also Def. 14.15.)

**Def. 14.13**     **Hate** is a willingness to lower the hekerger of another person, unconditionally.

In an immature individual love of the good and hate of the evil are equally moral

Oge-persons are here named after their originals, with the prefix *oge-*. Thus the oge-lover is one who loves the person of the ego and, being the most approving of the ego, is closest to it; it consists of mid-memories of the parents in a normal child, plus mid-memories of spouse and children in a normal adult. Similarly we may speak of oge-beloved, oge-family, oge-friends, oge-strangers, oge-foreigners, and oge-enemy.

The oge-enemy is least approving of the ego, and characterised by hate. In a normal child the oge-enemy is formed by repute rather than experience. It represents people or societies, other than the ego's own, which threaten harm or destruction to the ego's person or society. As such, it is the immoral part of the oge.

Because the ego is capable of both good and evil towards others, it also is polarised like the oge. We will call the good pole of the ego the **top ego** or **high ego**, and the evil one the **bottom ego** or **low ego**; this top and bottom are relative to the ego's subjective co-ordinate system, so that the top ego is above the ego's empirical world, and the bottom ego is below it. Thus the good is high and the evil is low.

By L.A.L. the polarised oge will match its polarisation to that of the ego: the oge will curl around the ego, with its oge-lover at the top and its oge-enemy at the bottom; so we may speak of **top** and **bottom**

**oge**, or **high** and **low oge**. The good oge-lover and the evil oge-enemy will thus be in the traditional locations, relative to the empirical world of the ego, of heaven and hell: above the blue sky and below the ground. This is a theological matter to be discussed later.

Once the oge has curled around the ego in this way, its middle portion will expand around the equator of the ego, thereby making the oge a hollow shell, concentric with the ego.

The equator of the oge is the most public part of it, since it represents the attitudes of society in general, while the poles, which represent the attitudes of particular individuals, such as parents and teachers, are the most private. The top oge also has the greatest quantity of mid-memories in it, relative to the equator and, in most people, relative to the bottom oge; thus the top oge, the good or moral part of the oge, is the most influential, by L.A.L., because of having the most hekergergy.

**Def. 14.14**     **Maturation** of agents is their hekergergy increase.

Maturation takes two forms: increase of summation hekergergy, and increase of emergent hekergergy. The former is addition of new parts — memories and beliefs — and the latter is rearrangement of existing parts into structures of higher hekergergy.

We have to suppose that the maturation of ego and oge is a two stage affair: if all memories went to either the ego or the oge on a basis of approval or disapproval, the ego would never have any memories of other people disapproving of its actions. So we assume that first stage growth of ego and oge occurs in this way, and that second stage growth starts when each agent is large enough to map its own copies of the other's mid-memories.

**Def. 14.15**     **Identification** occurs when an oge-person, or group of them, becomes joined to the ego.

It is the oge-lover with whom the ego first identifies. Identification allows us to explain the possibility of the ego loving, given the mind hekerger principle. Love, as the willingness to give unconditionally, requires a decrease of the hekerger of the ego — contrary to the mind hekerger principle: the decrease is the gift. But if the oge-beloved is a part of the ego, through identification, such giving is compatible with the mind hekerger principle: the ego is giving hekerger from one part of itself to another. (Identification is also called, in ordinary language, a bonding between two people; the term is avoided here because of its other use.)

**Def. 14.16**     **Inclination-duty conflict** is conflict between the ego and the public portion of the oge.

**Def. 14.17**     **Neurotic conflict** is conflict between the ego and private portions of the oge.

**Def. 14.18**     **Theoretical possession** or **ownership** is an emergent relation between a theoretical person or group of theoretical people, and a theoretical object, characterised by control of the object by the owner, and lack of such control by non-owners.

The relation of possession, and its emergent property of control, are imaged into the theoretical mind, such that the ego of the possessor says “Mine” of the empirical object, and the oge says “Yours.” The hekerger of a possession is a portion of the wealth of the possessor.

**Def. 14.19**     **Theoretical wealth** is the emergent relation between the hekerger of a theoretical object and a theoretical need of the possessor of the wealth.

**Def. 14.20**     **Material wealth** is the emergent relation between the hekerger of a material — that is, empirical — object and an empirical need of the possessor of the wealth.

We must digress briefly into the subject of wealth in order to discuss conflict between the ego and the oge.

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Material wealth is primarily the functionality, or utility, of material objects. Functionality, such as that of a bed, a spoon, a hat, a screwdriver, an automobile, food, or shelter, is one or more emergent properties of an object that enable the increase or maintenance of the hekerger of the user of the object — thereby fulfilling the need that is one of the terms of the wealth. Such functionality or utility is hekerger, hence has value.

Material wealth is either consumable, such as food and fuel, or non-consumable, such as furniture, tools and machines, and art objects. Some material wealth is based purely on hekerger — value — rather than on functionality or need, in that it possesses hekerger through rarity, such as rare stamps, or rarity plus beauty, such as gold or diamonds; rarity contributes to hekerger in that the rare is improbable

The wealth of material objects ceases to exist if the need that is one of its terms ceases to exist. Thus with the development of the automobile the proverbial need for buggy whips vanished, so that buggy whip factories that continued to produce buggy whips no longer produced wealth. And when the rare becomes abundant, as happened with aluminum, its value decreases.

Because needs are dispersed throughout a society, material objects will produce wealth only if their arrangement is changed so as to fulfill those needs. This is distribution of the material objects within society, a rearrangement that causes hekerger to emerge, the hekerger being wealth. Thus the production of wealth requires both the production of material goods according to needs, and their distribution to these needs. Trade is thus a part of the production of the wealth of a society.

Other social changes that produce wealth through hekerger increase in accord with a need are production and distribution of information, and services that process information.

Both the value of money and economic forces are relations in the theoretical world that emerge out of theoretical wealth. Inflation occurs when the value of money goes down, due to there being more money than wealth in an economy; and deflation is the opposite of inflation. Wealth is also produced or maintained by services that rearrange money or ownership, such as banking, stock-broking, and insurance.

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**Def.14.21**     **Duty is an oge-compulsion that requires the ego to increase or maintain the hekerger of society.**

Inclination-duty conflict is largely a conflict over material wealth. The inclination of the ego is to acquire wealth for itself, in accordance with its basic attitude of selfishness; the duty of the ego is to give wealth to society, in accordance with the moral attitude of the oge. Such conflict extends to work — which is hekerger increase or maintenance — in the form of the mutually exclusive goals of working-for-self or working-for-others.

With maturation of the ego and oge the conflict between inclination and duty diminishes. This is because maturation of both agents leads to more and more identification between the ego and the oge society to which it belongs, so that the ego is loving towards it, rather than dutiful.

Besides material wealth there is also mental wealth. This is primarily growth of the ego, for each individual concerned. In the very young it results from love, in that loving parents allow ego-memories to go to the ego rather than the oge, so making for a strong ego. The mental wealth of the ego is also increased by education, and by a wide

variety of experiences such as is produced by travel and by diversity in daily life.

Deprivation of love in the young leads to stunting of the ego, just as deprivation of food leads to stunting of the body. Such stunting of the ego produces neurotic conflict between the ego and oge. Three examples are inferiority and superiority complexes, compulsive failure, and sexual neurosis.

Inferiority and superiority complexes are prejudices about the merits of the ego, relative to other people. If the ego comes to believe that it is inferior then it will prejudicially select evidence in favour of this belief, and reject evidence against it, by L.A.L. The rejected evidence goes to form an equal and opposite prejudice in the oge.

**Def. 14.22**    An **ego inferiority complex** is a prejudice, in the ego, that the ego is inferior to its peers; an **oge superiority complex** is a prejudice, in the oge, that the ego is superior; an **ego superiority complex** is a prejudice, in the ego, that the ego is superior; and an **oge inferiority complex** is a prejudice, in the oge, that the ego is inferior.

The complex in the ego represents the individual's self-opinion, and the one in the oge represents the opinion that the ego believes others have of it.

Ego-oge conflict will result from the existence of these complexes because they produce mutually exclusive goals. A superiority complex has the goal of over-achievement by the individual and an inferiority complex the goal of under-achievement. Within the conflict a victory for the superiority complex is a compulsion: a compulsion to undertake some task in order to achieve. A victory for the inferiority complex is an inhibition: an inhibiting of action, thereby ensuring the failure expected by the inferiority complex.

The conflict is intensified by the fact that the generation of the complexes in early childhood is usually brought about by similar complexes in a parent, who is unloving in feeding its own complexes at

the expense of the child. Thus a parent with an ego inferiority complex will not only believe the child to be superior, but seek proof of this. Such selfish parental acts are unloving and thereby produce crisis feelings in the ego of the child, associated with the complexes.

**Def. 14.23**     **Ego compulsive failure** is an ego compulsion to fail, in defiance of a secondhand ambition; **oge compulsive failure** is an oge compulsion for the ego to fail, as a result of a parent denying the ego the opportunity to mature.

Secondhand ambition occurs when a parent has failed in their own ambition — to be a concert pianist, a doctor, a sports hero, etc. — and wants the child to be their surrogate achiever; the ambition then becomes an oge-ambition in the child. Such an oge-ambition denies the ego's own ambitions and so is unloving and a cause of crisis feelings. In later life the ego will thwart the oge in its attempts to achieve the oge ambition, and the oge will thwart the ego in its attempts to achieve its own ambition. Both thwartings are compulsive failures for the individual as a whole.

A parent who wants the child to remain dependent will withdraw love on occasions of independent achievement by the child, and supply it on occasions of failure. In later life the ego will seek needed love through failure, and shun success — aided in both by the oge.

**Def. 14.24**     **Sexual neurosis** occurs when some aspect of sexuality is driven to the oge-enemy by parents who regard it as evil.

Parental horror at the exhibition of infantile sexuality is unloving; the appetite, along with the parental association of evil with it, is driven to the bottom ego, where it matches, by L.A.L., the oge-enemy evil sexuality. The main effect of this kind of neurosis is to separate love and sex, so that sex with a beloved and love of a sexual

partner are impossible. Callous sex is characteristic of sex with a prostitute, and sex bonded to hate is characteristic of a rapist.

**Def. 14.25**     A **puritan** is one for whom all pleasure is evil.

Because of their neurosis, puritans cannot enjoy any normal pleasure. They usually hide from this by the ego compulsion of hard work, so that they have no time or energy for pleasure. Because they regard pleasure as evil, they seek oge help in fighting this evil — usually by legislation, such as Sunday-observance laws and Prohibition — and as a result are known variously as killjoys, spoilsports, wet blankets, and party poopers. Their rationalisation for their attitude is that a man's first duty is to God (which in this case is the upper oge — see Chapter 15), his second duty is to his fellow man (his oge), and only his third duty is to himself. Historically, in Europe, Protestants were more puritanical than Catholics, so the puritanical need to work became known as the protestant work ethic — a compulsion that perhaps explains much of the wealth of Europe and North America.

Cure of neurosis requires ego-oge co-operation in mutual restructuring. One essential feature of this is another person — a psychotherapist — or a group of people, who represent the oge and who help alter the oge by introjection. These oge representatives should ideally be loving, and must at least be sympathetic and patient. A second essential feature is the acknowledgement, by the ego, of the existence of the neurosis and a desire to cure it. And since the restructuring is difficult and painful, a third requirement is considerable effort by both ego and oge over quite a long period of time.

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**Def. 14.26**     An **ego-dominant type** is a person whose ego is stronger than their oge; an **oge-dominant type** is a person whose oge is stronger than their ego.

An ego-dominant type shows ego characteristics much more than oge characteristics: selfishness; vanity; immorality; ruthlessness; unconventionality; order-giving; freedom from guilt and shame, embarrassment, conscience, and political correctness; and a willingness to exploit the underprivileged. An oge-dominant type is unselfish, modest, moral, conventional, politically correct, obedient, susceptible to guilt, shame, embarrassment and conscience, and protective of the underprivileged.

Dominance is a matter of degree. Because L.A.L. is proportional to hekergy and inversely proportional to distance squared, oge-dominance results both from a strong oge and from an oge that is spatially close to the ego, while ego-dominance requires a weak or a distant oge.

**Def. 14.27**     **Rudeness ability** is an index of degree of ego-dominance.

An ego usually is able to be rude to someone of lower ego-dominance than itself (greater oge-dominance) and is unable to be rude to someone of greater ego-dominance.

Many social institutions formalise dominance with ranks in a hierarchy, as in feudal societies, the armed forces, business corporations, churches, universities, civil services, and trade unions. Higher ranks give commands to lower ranks and lower ranks obey higher ranks. In such organisations the most ego-dominant types rise to the top, to leadership positions, and the most oge-dominant types sink to the bottom, to servile positions. Higher and higher levels are characterised by increasing ruthlessness of competition for promotion.

Since the upper portion of the oge also represents the state, the government, the mother country, or the fatherland, individual political attitudes and beliefs will to some extent be determined by the oge. These vary on a spectrum from right to left, from reactionary to radical; and an individual's position on this spectrum is indicative of his or her position in the dominance scale. Oge-dominant people will be radical,

concerned for the good of society and all individuals in it, desiring government protection of all with a social safety net; ego-dominant people will want minimum government interference in their selfish entrepreneurial activities, which means minimum controls and minimum taxes. Since taxes are necessary for a social safety net, political disagreement is inevitable among people of different dominances.

Furthermore, this disagreement is due to ego-oge imbalance, and this in turn is due to L.A.L., which is essentially irrational. Thus most political dispute is irrational. Those in search of political sanity should consider that a social safety net is desirable for the health of society, just as are good universal education and good universal medical services, but such a safety net is expensive: only a society with considerable wealth can afford it — and such wealth is usually produced by ego-dominant types. As always, constructive compromise is needed between ego-dominance and oge-dominance.

**Def. 14.28**      **Elation** is the feeling the ego has when it wins an ego-oge conflict.

**Def. 14.29**      **Depression** is the feeling the ego has when it loses an ego-oge conflict.

**Def. 14.30**      **Insanity** is the result of extreme dominance.

**Def. 14.31**      A **psychopath** or **sociopath** is a person with little or no oge.

A psychopath has little or no moral sense and behaves morally only for prudential reasons; he has little or no feelings for others, so that he is capable of extreme callousness and ruthlessness; he has no experience of shame, guilt, or conscience. If he has ability he will become rich and powerful. Psychopathy is formed in early childhood by parents who, infatuated with their child, are so over indulgent that little or no oge forms.

**Def. 14.32**     A **schizophrenic** is a person with little or no ego.

A schizophrenic is a result of unloving parents and thus has a very weak ego and an unloving oge. The oge has great control over the ego in the early stages of ego-oge conflict, thereby producing depression, and threatens the destruction of the ego in later stages. The ego is well aware of this, but misinterprets it as an external threat. This misinterpretation is seen by others as delusion: persecution when “They” (or the government, or the police, or some other authority) are controlling me with invisible rays (or waves, or forces); and, later, “They”, or some group, are trying to kill me. The ego’s only defence is bluff: you mustn’t harm me because I am the Messiah (or some other notable) a bluff that is obviously a delusion of grandeur to others. The destruction of the ego means the destruction of the empirical world of the ego, since the former is part of the latter; this also may be interpreted as an external threat, so that the ego prophesises to all who will listen that the end of the world is coming. If the ego is destroyed by the oge, catatonia results; destruction of the whole individual by the oge is suicide. The ego may in desperation make a pre-emptive strike which also, through misinterpretation, is directed externally; this is homicidal mania, usually followed by suicide as punishment by the oge.

**Def. 14.33**     A **manic-depressive** is a person who oscillates between ego dominance and oge dominance.

A manic-depressive’s oscillations between ego-dominance and oge-dominance are experienced by the ego as elation and depression, and are externally characterised by delusions of superiority and inferiority.

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A variety of lesser features of human behaviour may be explained by means of the oge. They are here presented in alphabetical order.

**Def. 14.34**     **Archetypes** of the Jungian collective unconscious are oge-persons.

Jung was mistaken in supposing that there is only one collective unconscious mind, accessible to all; he committed the identity error when he inferred this from the similarity of archetypal symbols produced by a wide variety of people, but similarity does not entail identity: the public portions of oges have similar oge-persons by virtue of being public, but their contents are private by plurality.

Two exceptions to Def. 14.34 are the archetypes that Jung called the *persona* and the *umbra*: these are not oge-persons, but substructures of the ego.

**Def. 14.35**     **Astrology** is an erroneous attribution of the influence of the oge to the empirical planets.

The oge surrounds the ego, is beyond the blue sky, and has considerable influence in the personal fate of most people. As with the delusions of schizophrenia, this influence is usually attributed to the empirical world because of the ignorance of the existence of the oge. The prevalence of horoscopes in daily newspapers shows how widespread is the belief in astrology, while the absence of connection between the theoretical planets and the oge shows its futility. The belief in astrology naturally is most widespread among the oge-dominant.

**Def. 14.36**     **An authoritarian** is an ego-oge imbalanced person.

Authoritarians are order-givers (dominant authoritarians) or order-takers (subservient authoritarians), depending on the direction of their imbalance. Their attitude is that everyone must either control or be

controlled. They are authoritarians because they operate according to some authority — a code, a book, or a tradition, in the empirical world, but the oge is the authority in fact. Their authority makes them rigid and closed-minded: something is judged true or false according to their authority, rather than on its own merits; hence authoritarians are conservative and uncreative. All who reject their authority are their enemies. They are callous towards their inferiors and their enemies. They lack courtesy towards others, since courtesy is the result of caring for others; instead they behave according to a code or etiquette, which is an imitation, or algorithm, of courtesy. They perpetuate social hierarchies, and anyone who operates within a hierarchy by choice is probably an authoritarian. One form of authority particularly favoured by authoritarians because of its immutability is ancestry; thus authoritarians tend to be fascinated by pedigrees, and in the controversy over whether character is determined by nature or by nurture, they favour nature.

**Def. 14.37**      **Envy** is the ego's feeling of hekerger inferiority relative to some high hekerger in the oge; the high hekerger is the object of the envy.

**Def. 14.38**      **Extremism** is the attitude produced by great dissimilarity between oge-lover and oge-enemy.

Much of politics and of religion is an externalisation of the oge, and most extremism occurs in these areas. Because there is such a great dissimilarity between oge-lover and oge-enemy in an extremist, it follows that in a conflict situation he or she believes that the enemy is wholly evil, while “we” are wholly good and can do no wrong.

**Def. 14.39**      **A feeling of being watched** when committing a solitary guilty act is due to the oge watching.

Most people regard such a feeling as an illusion or a fancy, but it is a fact — although it is misdirected on to the empirical world. A later compulsion to confess is of course an oge compulsion, and shows that the oge was indeed watching, since it knows about the act.

**Def. 14.40**     **Gossip** is a process of oge communicating with oge.

**Def. 14.41**     **Hypnotism** is abdication of the control of the body, by the ego, in favour of the oge.

In a hypnotic situation the oge is represented by the hypnotist, who puts the ego to sleep and then gives orders to the oge for future control of the body. Because the oge is moral, it is impossible for a hypnotist to make a hypnotised individual behave immorally.

**Def. 14.42**     **Jealousy** is envy (Def. 14.37) of another's receipt of love.

**Def. 14.43**     Much of **magic** is an attempt to control the oge.

If the efficacy of magic is generally believed within a society then such things as the evil eye and love potions will work to some extent. A victim of the evil eye will not only believe in his doom, but so will his oge; and his oge may well then proceed to destroy him, in accordance with the will of the witch doctor, and the ego will resist little because it will believe resistance to be futile. A love potion may work in a similar manner: a reluctant maiden or an impotent lover, who are held in this condition by the oge, may have their oge attitude changed by knowledge of the administration of the potion.

Name magic is based on the belief that control of an agent is possible by means of their true name; so a demon — an oge person from the oge-enemy region of the oge — may be controlled if its name is known. This is not entirely false, since a true name is the name bonded to the agent, so that the demon might be brought into the empirical world, as a vision (Def. 14.50), by control of its name; the

belief in the magical power of the demon to increase the hekerger of the ego is of course false.

**Def. 14.44**     **Malice** is ego harm to the oge.

If the hekerger of the oge is diminished then the hekerger of the ego increases relatively. There is no actual increase of the ego's hekerger, so that the satisfaction, due to the relative increase, that is felt by a malicious ego is illusory. Directed materially towards the oge at large, malice is vandalism. Directed towards specific oge-persons, it is hurtful remarks, pity (which is compassion adulterated with malice), practical jokes, and anything else that lowers the hekerger of the person concerned.

**Def. 14.45**     **Mental health** is harmony between ego and oge.

Mental health in the young is characterised by absence of neurosis and absence of dominance. In the mature it is this, and also the absence of extremism and of malice, as well as considerable ego-oge identification, hence little inclination-duty conflict. Note that harmony is a relation, hence has hekerger and so is valuable; and Plato defined justice, both in a just man and in a just society, as harmony between the parts.

**Def. 14.46**     **Projection**, so called, is filtration of empirical perception through the oge.

Projection, such as the projection of an ideal on to a lover or a hero, and of evil on to an enemy, is the empirical perception of such material people through the corresponding oge-persons. This occurs because the oge is between the ego and the mid-person, so that the mid-person is perceived with some of the oge features — as an object seen through red glass is red. The mid-person will move, by L.A.L., to be opposite the corresponding oge-person, and the corresponding part of

the ego will be closest to the mid-person. If the mid-person is an oge-enemy the ego will see him as evil, because of filtration through the oge-enemy, and the ego will behave evilly towards him because the evil part of the ego will be opposite him. Equally, if the mid-person is sexually suitable then the oge sexual ideal — the Jungian *anima* or *animus* — will be the filter so that the person will be perceived as sexually perfect, and the corresponding part of the ego will behave accordingly: macho if a male ego, flirtatious if a female ego; the result is infatuation. Because the oge-persons that filter are prejudicial, much of such love and such hate is illusory.

Notice that projection is so called because to a realist the empirical person is real and outside the real head of the perceiver, so that the obviously subjective qualities perceived by that perceiver have to be projected somehow out of his head on to the material person. As we have seen, no such literal projection is possible because there is no mechanism for it; and in the light of the Leibniz-Russell theory of perception, it is unnecessary.

**Def. 14.47**     **A public ritual is a process of obtaining oge approval of the object of the ritual.**

It is because the ritual is public that the fact of it becomes introjected into the oges of all concerned. The most common public ritual is a marriage: a process of obtaining oge approval of two people having sexual relations and reproducing — as opposed to oge disapproval of living in sin and illegitimate birth. Marriages are often believed to be made in heaven, because that is the part of the oge that approves. Another kind of ritual is a coronation; because the oge is one of the meanings of the word God, as will be explained in the next chapter, a monarch who is publicly crowned may claim the divine right of kings. Other public rituals are rites of passage, such as baptisms, graduation from school and college, induction into a social group, and funerals.

**Def. 14.48**     **Self-sacrifice** is the sacrifice of the individual for the good of society.

Self-sacrifice is usually an oge action, not an ego action; it is an oge action because it is moral, and it is not an ego action because it requires ego diminution of hekerger, which is contrary to the mind hekerger principle. Such self-sacrifices clearly are performed by oge dominant types. The exception to this is when the sacrifice is motivated by love, in which case there is ego identification with the oge-person or oge-persons who benefit from the sacrifice.

**Def. 14.49**     **Sleepwalking** is a state in which the ego is in a deep sleep and the conscious oge is controlling the body.

**Def. 14.50**     **Visions** are manifestations in the ego's empirical world that originate in the oge.

Although the name implies otherwise, visions may be perceived through other senses than sight — as with hearing voices. The most usual visions are ghosts and religious visions. Ghosts are visions of deceased oge-persons. Religious visions are usually messengers from the oge-lover, in its capacity as one of the meanings of the word God; they usually take the form of hearing voices giving moral commands, although there may be a visual messenger, such as an angel, as well.

## 15. Gods

This theory of mind provides six meanings for the word God, all of which conform in one or more ways to the traditional uses of the word. They are:

- Def. 15.1      The **oge-god** is the oge.
- Def. 15.2      The **holy ghost** is the mind hekergy principle.
- Def. 15.3      The **deified teacher** is the oge-person of a revered religious teacher.
- Def. 15.4      The **panacea god** is a prejudice.
- Def. 15.5      The **philosopher's god** is the theoretical world.
- Def. 15.6      The **psychohelios**, a third agent that is defined in the next chapter, is the god of the mystics, the god of truth.

We consider each of these in turn.

As an agent the oge-god is either an oge-person or a set of oge-persons, depending on whether the oge is unified or not; as such, it is either a personal god, or a pantheon of personal gods. Unification of oge-persons into one agent is an increase of configuration hekergy of the oge, so the development of monotheism in religion was a moral advance. This is one reason for the theological denial of Manichaeism, which held that there are two gods, one good and one evil: that is, the good, or high, and evil, or low, portions of the oge are separated. Because each person's empirical world is entirely within their ego, and the oge is beyond the ego, the oge-god is beyond the blue sky and so

transcendent to that empirical world and all the empirical people in it; so it is one god for all these empirical people, but it is not one god for the theoretical people of which these empirical people are images: each theoretical person has their own, numerically distinct, oge-god, and all of these are largely public within supposedly one tribe or culture. The publicity is, of course, publicity by similarity, not publicity by identity, so that to suppose that all these similar oges are one is to commit the identity error. Because of the influence that the oge-god may have upon the empirical world of the ego, the oge-god is immanent to the empirical world as well as transcendent to it. Because of its basic attitude the oge-god is a god of morality, issuing proscriptions and prescriptions. Being an agent, the oge-god has human attributes: it may be loving of the ego, and to some extent merciful, just, forgiving, vengeful, etc. In oge-dominant people the oge demands submission by the ego; such submission is worship of the oge-god. Communication to the oge-god takes the form of prayer, and some kinds of prayer are within the oge-god's power to grant: prayers for aid in overcoming neurotic impediments to moral works, for example. Other kinds of prayer are quite beyond the oge-god's power to answer, such as prayers for rain — because their outcome depends upon the theoretical world, not the oge. The oge-god may also be cowed temporarily by taboo words, as in swearing and blasphemy. Oge dominant people are far more likely to believe in the oge-god than are ego dominant people, since they are more likely to have religious experiences of it — because of its strength and proximity. Since the oge is irrational, apart from introjected rational beliefs, so is the oge-god; as such it is far from perfect. And it is definitely finite, so is not omnipotent, omniscient, all-loving, etc.

The mind hekergy principle is the source of creativity (Def. 16.1) in the theoretical mind. As such it may be called the Holy Ghost. It also is transcendent and immanent.

The deified teacher is an oge-person, such as Christ, or Budda. If the oge is unified, this god is one with the oge-god. Along with the mind hekergy principle we thus get the trinity of traditional christianity.

There is a sense in which it is true to say that the trinity is one, in that the real world is a unity and the three gods of the trinity are parts of it; but this is largely logic-chopping, caused by the desire to maintain monotheism: the intensional number three cannot be equal to one.

The panacea god is the god rightly denied by atheists: it is a belief that is a mere prejudice, an incoherent mixture of the ideas of the other five gods. It is an imaginary anthropomorphic being which is used as panacea explanation, refuge from ignorance, solace in the face of fear of the unknown, support of prejudices repairing human injustices and mortality, destroyer of curiosity, opiate of the people, and rocklike authority for the closed minded. There are no grounds for believing it to exist, and, given our knowledge of science, many grounds for denying its existence: so it has nominal meaning only. Atheists are among those who believe, mistakenly, that this is the only meaning for the word God.

The theoretical world is, philosophically, the most rational meaning for the word god. If we use old-fashioned language, then as cause of the birth and growth of the individual mind it is the creator of the soul; as cause in theoretical perception, it is the continuous creator of the empirical world of each consciousness, and as such the ground of all empirical being; as the totality of theoretical causes it is all powerful; as a finite but unbounded space-time it is, in this limited sense, infinite; being of maximum possible wealth it is perfect; since its top relation has intrinsic necessary existence it is *causa sui*, self-caused, and thereby self-explanatory; as the first step in the entire chain of extrinsic necessary existence its top relation is first cause; being beyond the blue sky it is transcendent, and so radically empirically imperceptible; since each empirical world is part of a theoretical mind which is part of the theoretical world, it is immanent to each empirical world; and it is wholly rational in the sense of being of maximum hekergy. However it is not a person, so it does not have mental attributes such as love or mercy and it is indifferent to prayer and sacrifice. Note that Aristotle defined god as pure form (that is, pure relation, in the present context) beyond heaven; Plotinus said that out of

god (the theoretical world) emanated *nous* (theoretical mind), out of which emanated soul (ego), out of which emanated matter (empirical world); Spinoza's god is the theoretical world; and St Anselm originated the ontological argument to prove the existence of God.

## 16. *Rational Mind*

**Def. 16.1**      **Creativity** is the talent for novel, original, hekerger increase.

Creativity is a product of the mind hekerger principle.

In mathematics creativity may produce an axiom set, a conjecture, or a proof. This may be social or individual: when first made it is individual and when it is published it becomes social, while if it is made in ignorance of a previous publication it is individual. Equivalent hekerger increases occur in individuals when they are being taught or are studying, but these are not a result of creativity because not original.

In science creativity is creation of theories and design of experiments.

Creativity is possible wherever hekerger increase is possible: in mathematical invention, in design of experiments, in invention of theories, in writing of poetry and literature, in composing music, in sculpting, painting, invention of new technology, design of buildings and gardens, cooking, etc.

**Def. 16.2**      **A genius** is anyone with exceptional creativity.

We earlier defined the irrational as any ideas ordered by L.A.L., while the rational is any ideas ordered with maximum hekerger. Ideas ordered by L.A.L. have emergent hekerger part way between those of chaotic orderings and rational orderings.

By the mind hekerger principle the ego will try to increase its rationality if it can. The possibility of this comes from the fact that the similarity of ideas ordered by L.A.L. is only fadingly transitive, so that ideas at the periphery of a large ego may be fairly dissimilar to the ego-memories at the core of the ego. Thus by suitable attention the ego may arrange these ideas other than by L.A.L., and will do so when the

rearrangement has a greater hekerger. The ideas are, of course, abstract ideas — relations in the mind, intensional meanings — and their rearrangement is thought.

**Def. 16.4**      **Intelligence** is the capacity to rearrange ideas rationally.

Apart from lack of intelligence, the main obstacles to rational thought and rational feeling are subjectivity and prejudice.

**Def. 16.5**      The **psychohelios** is that part of the ego that is wholly rationally ordered.

As it grows the psychohelios increasingly becomes an agent in its own right, distinct from the ego. As such it is conscious and able to act, although its consciousness and its ability to act are limited by the fact that its attention is invariable. Being of maximum hekerger, the psychohelios has no equivalent arrangements of its parts: in it, the value of  $e$  in  $\ln(t/e)$  is one, so it is little able to manipulate ideas, including motor ideas. But it can rearrange ideas that are present to it, by its overall field, in accordance with the mind hekerger principle. Its overall field also has an influence on the ego, making it desire to enlarge the psychohelios; this is an act of love on the part of the ego.

The psychohelios is the sixth feasible meaning of the word god.

Because its emergent hekerger is the maximum possible, the psychohelios is similar to the theoretically real world, which is of maximum possible hekerger, the best of all possibles. This similarity is of course truth: similarity truth. The psychohelios, although it does not have as great a density of detail as the real world, is absolutely true in all the detail that it does have.

**Def. 16.6**      The **suprarational** is the state of a theoretical mind that has maximum possible emergent hekerger.

The suprarational is the state in which the theoretical mind is all psychohelios; the state that Plato called wisdom, Spinoza called *scientia intuitiva*, happiness or blessedness, Hegel called the sublation of the finite self into the infinite absolute, and the mystics call union with god.

In the suprarational state all irrationally ordered ideas have either been reordered into the psychohelios, or else discarded. This means that both the ego and the oge have been reassembled into the psychohelios; as the mystics might put it, the death of the self is followed by the rebirth of consciousness in union with god. This is consciousness of absolute truth, absolute beauty, and absolute goodness. Although its occurrence is *post mortem* relative to the ego, it is *ante mortem* relative to the theoretical individual. The death of the self, or ego, also requires the end of the world — the empirical world, of course, not the theoretical or real world.

**Def. 16.7**      The **ethical** is anything that moves the individual towards the suprarational.

The ethical, in its present stipulative sense, is quite distinct from the moral: the moral is oge hekergey increase and usually irrational, while the ethical is psychohelios hekergey increase and rational. For this reason the ethical is never proscriptive or prescriptive, since these, in being moral, are unethical. Thus the ethical cannot be taught: to teach the ethical is to have the oge demand diminution of the ego in favour of the psychohelios, and this is too similar to ego-oge conflict to have any chance of success. So the ethical cannot be disciplined by others, it must be self-disciplined.

But although the ethical cannot be taught, it can be explained and inspired. The inspiration comes from the fact that by definition the suprarational state is the most valuable achievement possible for anyone.

There are three stages in the ethical progress towards the suprarational: maturation of ego and oge, growth of the psychohelios, and diminution of the ego and oge.

Maturation of ego and oge is necessary for maximum identification of the two. Without this, diminution of the ego would not include diminution of the oge, so that the oge could triumph over the ego. Such maturation constitutes mental health, for which absence of neurosis, dominance, and extremism are requirements.

Growth of the psychohelios is promoted most by wide-ranging curiosity about reality. This is aided by intelligence, creativity, a good education, access to libraries, leisure to think, significant conversation, ego maturity, and exclusion of extensional and nominal meanings in favour of intensional mathematics.

Diminution of the combined ego and oge is, on the part of the ego, a giving of self to the psychohelios, which giving is love of god; and, on the part of the oge, it is an acceptance of this, based on a culture of religious freedom.

We cannot know in advance what the experience of the suprarational would be like, for two main reasons. One is that ordinary language is fitted for the irrational and, to a lesser extent, the rational, so that it is largely useless for describing the suprarational; this is why mysticism is mystical — rather than mystification, as sceptics like to believe. The second is that a mind in a state of maximum hekerger must have an emergent unifying relation, and by the principle of novel emergence this relation must have novel emergent properties unknowable at the rational level.

However, some idea of the strangeness of the suprarational, and of how far removed it is from everyday life and common sense, comes from consideration of the illusions of irrationality, of which there are at least ten. We cannot manage without them in practical daily living, but the suprarational is not a part of that. They are:

1. Because the theoretically real world is of maximum hekerger it is unified by an emergent relation, so that any experience which separates parts of it from one another must exclude this unity. The

major separation in irrational experience is the division of everything into the Me and the Not-Me. This is an error, and leads to the illusion of individuality: the Me is the ego which, as an irrational structure, is dissimilarity false. Individual people are not really individual at all, they are one with the real world and only have an illusion of individuality.

2. Because of this, individual death is an illusion. What we call an individual person is an image of a world-line that has a beginning and an end in time, and we call these latter birth and death. But these are only hekergergy changes within the unified totality, in accordance with the principle of conservation of hekergergy, and as such part of the perfection of the whole.

3. Everything concrete is illusory: all secondary qualities, such as colours, sounds, tastes, smells, and feels are ego reactions to mid-qualities manufactured by the theoretical sense organs. These are crude representations of complex theoretical structures, necessary in theoretical perception for the empirical perception of relations, since relations cannot exist without terms; but their very crudeness makes them illusory. They are the illusion of matter.

4. This means that one's own empirical body is an illusion that ceases to exist in the suprarational state; consciousness without the body is what the mystics call ecstatic. The entire empirical world also vanishes in this state — because the ego has ceased to exist.

5. With the disappearance of the empirical body goes the illusion of the subjective co-ordinate system that has “I, here, now” as its origin. There are no real co-ordinate systems. Consciousness without a co-ordinate system is what the mystics call eternal life: it is timeless rather than infinite time.

6. Since the theoretical world is the best of all possibles, the one that has maximum possible hekergergy, it has no equivalent arrangements; nothing in it can be other than what it is, everything is fully determined. Thus chance is an illusion, due to ignorance of real causes. So possibility relations (Def. 1.12) are either necessities or else purely nominal relations.

7. Because of 1 and 6, freedom of the will must be an illusion. This is freedom as usually understood: “I” am free to act as I choose, and any such decision to act is the first cause in a new causal chain. In a world of maximum hekerger, such causal beginnings are not possible. (There is another meaning of freedom, due to Spinoza, which is not illusory. Spinoza distinguished *actions*, in which the ego is an agent and free; and *passions*, in which the ego is a patient and unfree. An action is behaviour whose cause is internal to the agent, and a passion is behaviour whose cause is external to the patient. For Spinoza control of the passions is the means to the suprarational.)

8. Good and evil, in the moral sense of oge judgements, are illusory. This does not mean that the social hekerger increases that we call good do not occur, nor that the decreases that we call evil do not occur; they do occur, and are part of the perfection of reality. But our colourings of the images of them as good and evil are illusory.

9. Our sensation of passage of time must be an illusion: since time is a dimension of space-time, nothing travels along it and there is no passage.

10. There are no intrinsic sets in the real world, and no contingent sets, so all classifications by means of them are illusory. Intrinsic sets are unreal because of the sheer quantity of similarities needed for them, which almost certainly do not exist, in accordance with Occam’s Razor; and contingent sets do not exist because their unities are only nominal. We think in set-theoretic ways because we form sets by L.A.L., but this is irrational and false. However, certain extrinsic intensional sets exist, such that each member is intrinsic to the set and extrinsic to the intension of the set: term sets, property sets, sets defined by the extrinsic intension of being within an actual closed boundary, or by being compound relations, or by being wholes.

\* \* \*

In conclusion, because of the nature of intensional mathematics, of the psychohelios, and of the theoretical world, we can say that

mathematics, theoretical science, metaphysics, ethics, and theology are ultimately all the same quest for suprarational truth. And all of them, if properly done, are like the present work in having cascading emergence of definitions and explanations — in having intensional meaning.

# REFERENCE

## *Glossary of Symbols*

*The symbols are listed in order of original appearance.*

A, B, C, etc.: relations  
a, b, c, etc.: terms of relations.

$av(c_1|c_2|...c_n)$ : a possibility  
relation having  
antecedent a and  
consequents  $c_1, c_2, ... c_n$ .

AqB: A necessitates B.

ArB: B necessitates A.

ASB: A necessitates B and *vice versa*.

Q, R, S: absence of necessity.

t: similarity.

T: dissimilarity.

A, B, C, etc.: sets.

R: the term set of the relation R.

A, B, C, etc: intrinsic property  
sets.

l: set-membership.

$\{A(RT)\}: \{x: xRT\}$ .

=: set identity.

f: intersection.

m: commonality.

g: union.

n: coupling.

i: subset.

k: either subset or set identity.

h: superset.

p: subintension.

o: superintension.

MS: the commonality of the  
members of S

A': extensional complement of  
A

A': intensional complement of  
A

IqEqN: intensional meaning  
entails extensional  
meaning which entails  
nominal meaning

U: truth-functional negation

z: truth-functional conjunction

y: truth-functional disjunction

h: truth-functional implication.

1: truth-functional equivalence

&: intensional conjunction

A, B, C: wholes

<: part of

>: has a part.

u: intensional truth.

U: intensional falsity.

g: the greatest intensional

number

$t^n$ : degree of similarity  $n$ .

$T^m$ : degree of dissimilarity  $m$ .

$l^n$ : degree of membership  $n$ .

$L^m$ : degree of non-membership  
 $m$ .

$u^n$ : degree of truth  $n$ .

$U^m$ : degree of falsity  $m$ .

$S_R$ : entropy of  $R$ .

$H_R$ : hekerger of  $R$

$G$  : the wealthiest system

$A$ : the least axiom set of  $G$

$T$  : the top relation of  $G$

$C$  : a contingent system

$R$  : the real world

## Glossary

*Italicised words are words listed under their own headings in this Glossary.*

ABSOLUTE VALUES: *Theoretical values, actual hekergeries.*

ABSTRACT IDEA: *A relation or an intensional meaning which exists within a theoretical mind.*

ACTION BY THE EGO: *The control of the theoretical body by means of motor-ideas. See also willing by the ego.*

ACTION-POINT: *A point in the theoretical mind, at which a motor-idea is delivered to a set of efferent nerves.*

ADDITION: *The addition of two intensional numbers,  $m$  and  $n$ , is the intensional binary operation, or intensional function, having the two of them as its argument and their sum as its value.*

ADICITY: *A primitive property of a relation; meta-linguistically, the number of terms that a relation has.*

AGENT: *Anything which has some awareness of, and some control over, its environment.*

ALLOWS: *If A is the antecedent of a possibility relation we say that A allows each of the consequents of that possibility relation.*

ANGLE SEPARATOR: *A separator that is a right angle and is symmetric.*

ANTECEDENT OF A POSSIBILITY RELATION: One special term of a *possibility relation*; all its other terms are called *consequents*. In some cases the antecedent may be two or more terms, as with a binary operation, for example.

APPROVAL: Evaluation of an *action* as *hekergy* increasing, as opposed to disapproval, which is evaluation of it as *hekergy* decreasing.

ARBITRARY: The *random*, some *degree of contingency*.

ARCHETYPES: (of the Jungian collective unconscious) are *oge-persons*. (*Persona* and *umbra* excepted.)

ARRANGEMENT: If a *unifying relation*  $R$  has a *term set*  $R$  then the arrangement,  $A$ , of  $R$  is the *compound relation* consisting of the set of every *compoundable relation* between any *members* of  $R$ .

ASTROLOGY: An erroneous attribution of the influence of the *oge* to the planets.

ASYMMETRIC: A dyadic relation is asymmetric if it has a mathematical sense, as a vector has sense, and it is symmetric if it does not have a sense. A dyadic *relation*  $R_{ab}$  is asymmetric if it is *dissimilar* to its *inverse*,  $S_{ba}$ .

ATOMIC DURATION: A *separator* that has unit duration, and is asymmetric. Also called a *temporal separator*.

ATOMIC IDEA: A neural switching, an On or an Off, in a *theoretical* brain.

ATOMIC LENGTH: A *separator* that has unit length and is symmetric; also called a *linear separator*.

ATOMIC VECTOR: A *separator* that has length, like a *linear separator*, and also unit mass, through having a length slightly less than that of a *linear separator*; it has a sense, and so is *asymmetric*.

ATTENTION: The *ego*'s focussing of its *consciousness*.

ATTITUDE OF THE EGO: The effect of the *ego*'s permanent structure on its *consciousness*.

AUTHORITARIAN: A person who is either significantly ego-dominant or significantly oge-dominant.

AVERSION AND DESIRE: The result of the *mind hekergy principle* acting on the *recognition* of the *possibility* of *pleasure* and *pain*.

AWARENESS BY THE EGO: The presence of *empirical objects* within the *ego*; they are the objects of this awareness. Also called *consciousness*.

AXIOM GENEROSITY: The cornucopia of definitions and theorems that a good axiom set produces; it is possible only with *intensional meanings*.

**AXIOM STRUCTURE:** Any *structure of relations* that produces *cascading emergence*, hence has *axiom generosity*.

**BELIEF, BY THE EGO:** A *proposition* that is incorporated into the structure of the *ego*, by *L.A.L.*

**BIPOSSIBILITY:** Any *possibility relation* having a *degree of possibility* of two.

**BONDING:** Two *theoretical ideas* are bonded when they are permanently joined together.

**BOTTOM EGO:** The portion of the *ego* below the bottom of the *empirical world*, relative to the *subjective co-ordinate system*; the *umbra*.

**BOTTOM OGE:** The portion of the *oge* below the bottom of the *empirical world*, relative to the *subjective co-ordinate system*; the *oge-enemy*.

**BOUNDARY:** A set of contiguous *dissimilarities*.

**BOUNDARY MAPPING:** A *mapping of boundaries*.

**CARTESIAN PRODUCT:** The Cartesian product of two sets *A* and *B*, symbolised  $A \times B$ , is the set of all *ordered sets* (a,b) such that  $a \in A$  and  $b \in B$ :  $A \times B = \{(a,b): a \in A \ \& \ b \in B\}$ . The definition may be extended to any number of sets.

**CALCULATION:** Algorithmic manipulation of *words* without their associated *abstract ideas*, in accordance with rules derived from *ordinary thought*. See also *pure thought*.

**CASCADING EMERGENCE:** The *emergence* of *relations* at many *levels*, those of one level being the *terms* of those emerging at the next level.

**CHANGE:** A *dissimilarity* compounded with a *duration*. Also called an *event*.

**CLASSIFICATION:** The process of collecting similar theoretical ideas into sets, by *L.A.L.*

**COMMONALITY OF PROPERTY SETS:** The commonality of two *property sets*, S and T, symbolised by  $SmT$ , if it exists, is such that every *member* of  $SmT$  is *similar* both to a member of S and to a member of T.

**COMMONALITY OF THE MEMBERS OF A SET:** The *commonality* of the *properties* of all of the *members* of an *intensional set*.

**COMMON SENSE REALISM:** The *belief* that *empirical reality* is *theoretically real*. Also called *realism*.

**COMPARISON:** A *bipossibility* relation having any pair of *relational properties* as its *antecedent* and *similarity* or *dissimilarity* as the *consequent*.

COMPLETE DISJUNCTION: An intensional *commonality*, or disjunction, whose *extension* is a *complete*, or *necessary*, set.

COMPLETE SET: Another name for an *intensional set*, any one of which is complete because defined by the *function every*; a complete set is a plurality unified by a relation. Also called a *necessary set*.

COMPOSITIONAL EXISTENCE: One kind of *extrinsic necessary existence*; *emergence*; the *existence* of a *term set R*, of a *relation R*, and a suitable *arrangement* thereof, necessitate the existence of the relation R. See also *distributive existence*.

COMPOSITIONAL PROPERTY: A relational property C is a compositional property if, given that any term of a relation R possesses C, then so must R.

COMPOUNDABLE RELATION: Any *kind of relation* that has one or more *properties* which are also possessed by the relation which unites a *set of instances* of that kind. For example, a set of contiguous lengths has a length.

COMPOUND RELATION: An *extrinsic intensional set* of *compoundable relations*, or an extrinsic set of extrinsic sets of them, or any higher *level* of extrinsic set of compoundable relations.

CONCEPT: A *word bonded* to an *abstract idea*.

CONCRETE IDEA: An *empirical memory* of a *concrete quality*, or a structure thereof.

CONCRETE MEANING: The concrete meaning of a *word* is *extensional*-any one *member* of that class of *concrete* memories which is the meaning of the word.

CONCRETE NAME: A *word* bonded to a *concrete* meaning.

CONCRETE QUALITY: Any *empirical* sensation.

CONFLICT: What occurs when there are two *agents* having mutually exclusive *goals* in a common situation.

CONJUNCTION THEOREM:  $\{A(A \cap B)\} = \{AA\}f\{AB\}$ , (6.7).

CONSCIENCE: A *feeling* produced in the *ego* by the *oge*.

CONSCIOUSNESS OF THE EGO: The presence of *empirical* objects within the *ego*; they are the objects of this consciousness. Also called *awareness*.

CONSEQUENT OF A POSSIBILITY RELATION: Any *term* of a *possibility relation* other than its *antecedent*.

CONTINGENCY: Any *possibility relation* having a *degree of possibility* greater than one, a plural possibility.

CONTINGENT FUNCTION: A *contingent set* of assignments of unique values to every *member* in an *extensional set* called the domain of the function which set is the extensional set of its arguments, the values being members of an extensional set called the co-domain or range of the function.

CONTINGENT SET: Any plurality of *members* which is not unified by a *relation*, and so is a many rather than a one; a plurality that has no *intension*; an *extension* which can be *enumerated* but which cannot be specified by a rule. Also called an *incomplete set*.

CORRESPONDING MEMBERS: See *degree of similarity of two sets*.

COUPLING OF TWO PROPERTY SETS: The coupling of two *property sets*, S and T, symbolised by  $S \cap T$ , is such that every *member* of  $S \cap T$  is *similar* either to a member of S or to a member of T, inclusively.

CREATIVITY: the talent for novel, original, *hekerger* increase.

DECOUPLING: The decoupling of two *property sets*, S and T, which are not *disparate*, symbolised  $S - T$ , if it exists, is the set consisting of those *members* of S which are not similar to any member of T.

DEGREE OF CONTINGENCY: The *degree of possibility* of a *contingency*.

DEGREE OF DISSIMILARITY: The degree of dissimilarity,  $m$ , between two *relations* A and B, symbolised  $A \nabla^m B$ , when  $A \nabla^n B$  (that is, the *degree of similarity* between A and B is  $n$ ), is  $m = (d+a)/(s+d+a) = 1-n$ , where  $s$  is the number of *properties* that one relation has which are *similar* to properties in the other,  $d$  is

the number of properties which are *dissimilar*, and if one relation has more properties than the other, the number of the excess is  $a$ .

**DEGREE OF MEMBERSHIP:** If  $A \text{t}^n B$  (that is, the *degree of similarity* between  $A$  and  $B$  is  $n$ ) then the degree of *set-membership*,  $l^n$ , of  $A$  in the *similarity set*  $\{A(tB)\}$  is  $n$ . This is symbolised by  $Al^n\{A(tB)\}$ .

**DEGREE OF POSSIBILITY:** The number of *consequents* of a *possibility relation*.

**DEGREE OF SIMILARITY:** The degree of similarity,  $n$ , between two *relations*  $A$  and  $B$ , having *property sets*  $A$  and  $B$ , symbolised  $A \text{t}^n B$ , is the *ratio*  $n = s/(s+d+a)$ , where  $s$  is the number of properties that one relation has which are *similar* to properties in the other,  $d$  is the number of properties which are *dissimilar*, and if one relation has more properties than the other, the number of the excess is  $a$ .

**DEGREE OF SIMILARITY OF TWO SETS:** Given two sets,  $A$  and  $B$ ,  $A$  smaller than  $B$ , and any ordering of  $A$ , then each *member* in this ordering may be matched with a member of  $B$ , such that no member of  $B$  is matched more than once. This matching thus produces a *contingent function* from  $A$  to  $B$ , and between each *argument* of this function and its *value* is a *degree of similarity*. The average of these degrees of similarity for every argument of the function is a *ratio*,  $r$ , say. There is then some *intensional function* from  $A$  to  $B$  that maximises  $r$ . Any pair of an argument of this intensional function and its value is defined as a pair of corresponding members in the two sets, and the maximal value of  $r$  is the degree of similarity of the two sets.

DEGREE OF TRUTH: If A is a representation of B and  $A \sim^n B$  (that is, the *degree of similarity* between A and B is  $n$ ) then the degree of truth,  $u^n$ , of A, relative to B, is  $n$ . This is symbolised  $u^n A$  if A is an *abstract idea* and B is a corresponding part of reality.

DEIFIED TEACHER: The *oge-person* of a revered religious teacher.

DEPRESSION: The feeling the *ego* has when it loses an ego-oge *conflict*.

DESIRE AND AVERSION: The result of the *mind hekergy principle* acting on the *recognition* of the possibility of *pleasure* and *pain*.

DIFFERENCE: If an *intensional natural number*  $m$  is *greater than* another,  $n$ , the difference between them, symbolised  $m - n$ , is the *adicity* of the *set difference* of their term sets,  $M - N$ .

DISCRIMINATION: Consciousness of the results of special mappings: see *patch mapping*, *boundary mapping*, and *scale mapping*.

DISJOINT: If the *intersection* of two *intensional sets*  $S$  and  $T$  does not exist,  $S$  and  $T$  are said to be *disjoint*.

DISJUNCTION THEOREM:  $\{A(A \sim B)\} \cup \{A \sim g\{AB\}\}$ , (6.9).

DISPARATE: If the *commonality* of two *property sets*, S and T, does not exist, S and T are said to be *disparate*.

DISSIMILARITY: A primitive *relation* which, like *similarity*, has *properties of relations* as its *terms*; the *intensional complement* of *similarity*.

DISSIMILARITY FALSITY: If a *structure* is a copy, representation, or reproduction of another, and they are *dissimilar*, then their dissimilarity is called the *intensional falsity* of the copy, relative to the other, or original. Also called *intensional falsity*.

DISTRIBUTIVE EXISTENCE: One kind of *extrinsic necessary existence*; the *existence* of a *relation* R extrinsically necessitates the existence of its *term set* R, and this is distributive existence. See also *compositional existence*.

DISTRIBUTIVE PROPERTY: A relational property D is a distributive property if, given that a relation R possesses it, then so must each term of R; and if R is a *top relation* of a *whole*, then so does each *element* of the whole.

DIVISION: The division of two *intensional natural numbers*, *m* and *n*, such that  $m > n$ , symbolised  $m/n$ , is the repeated *subtraction* of instances of *n* from *m* until no further subtraction is possible; the number of subtractions, *p*, is the quotient of the division, such that  $m/n = p$ . If after the *p* subtractions there remains a number *q*, then *q* is the remainder.

DOMINANCE See *ego-dominant type* and *oge-dominant type*.

DUTY: An *oge-compulsion* that requires the *ego* to increase or maintain the *hekergy* of society.

EGO: A *structure* of *ego-memories*, mutually attracted by *L.A.L.* because of their common feature of an *ego-memory*, which is a *mid-memory* of the *mid-body*.

EGO-COMPULSION: The forcing, by the *ego*, of an action against the *will* of the *oge*.

EGO COMPULSIVE FAILURE: An *ego-compulsion* to fail, in defiance of a secondhand ambition.

EGO-DOMINANT TYPE: A person whose *ego* is stronger than their *oge*

EGO INFERIORITY COMPLEX: A *prejudice*, in the *ego*, that the *ego* is inferior to its peers.

EGO-INHIBITION: The prevention, by the *ego*, of an action *willed* by the *oge*.

EGO-MEMORIES: *Mid-memories* that contain a *mid-memory* of the *mid-body*.

EGO SUPERIORITY COMPLEX: A *prejudice*, in the *ego*, that the *ego* is superior to its peers.

ELATION: The *feeling* the *ego* has when it wins an *ego-oge conflict*.

ELEMENT OF A WHOLE: Any one of: the *top*, or *unifying relation*, *R*, of the *whole*; any *member* of the *term set*, *R*, of *R*; any member of its *arrangement*, *A*, of *R*; or any one of these of a subordinate whole of the whole defined by *R*.

EMBARRASSMENT: A *feeling* produced in the *ego* by the *oge* as a result of the breaking of a taboo, or of a solecism.

EMERGENCE: The coming into existence of a *relation* and its *properties*. See also *submergence*.

EMERGENT HEKERGY OF A WHOLE: The *hekergy* of the *top*, or *unifying relation*, of the whole. See also *summation hekergy* of a whole.

EMERGENT LEVEL OF A RELATIONAL PROPERTY: The lowest *level* at which that *property* may *emerge*.

EMERGENT STRUCTURE: The emergent structure of an *axiom structure* is the *complete set* of relations *cascadingly emergent* from it.

EMPIRICAL: Anything known through the senses. See also *theoretical*.

EMPIRICAL CAUSATION: *Exclusively extensional functions* among *empirical* data, characterised by *universality*.

EMPIRICAL CAUSES: The arguments of *empirical causations*.

EMPIRICAL EFFECTS: The values of *empirical causations*.

EMPIRICAL MEMORY: The *ego's consciousness* of a *mid-memory*.

EMPIRICAL OBJECT: The *L.A.L.* reaction in the *ego* to a *mid-object*, hence an image of that *mid-object*.

EMPIRICAL PERCEPTION: Perception as we know it in everyday experience. See also *theoretical perception*.

EMPIRICAL REALITY: All that we empirically perceive around us that is potentially universally *public by similarity*.

EMPIRICAL SENSATION: The *L.A.L.* reaction in the *ego* to a *mid-sensation*; it is an image of a *mid-sensation*.

EMPIRICAL VALUES: The *ego's consciousness* of *absolute values*, distorted by subjectivity. Also called *human values*.

EMPIRICAL WORLD: The empirical world, of any one person, at any one time, is all that they *empirically perceive* at that time, other than introspectively; it is a *structure of empirical objects*, an image of a *mid-world*.

ENTROPY OF A RELATION: A measure of the entropy,  $S_R$ , of a *relation*,  $R$ , is the natural logarithm of the *probability* of  $R$ ,  $\ln(e/t)$ . See also *hekerger*.

ENUMERATION: A list of the names or descriptions of every *member* of a *set*.

ENVY: The *ego's feeling* of *hekergy* inferiority, relative to some high *hekergy* in the *oge*; the high *hekergy* is the object of the envy.

EQUAL: Two *intensional natural numbers*  $m$  and  $n$  are equal, symbolised  $m=n$ , if their relations  $M$  and  $N$  are *equiadic*.

EQUAL RATIOS: An *intensional ratio*  $a:b$  is equal to another ratio  $c:d$  if and only if  $ad=bc$ .

EQUIADIC: Two *relations* are *equiadic* if there exists at least one relation that is a *representative instance* of each of their *adicities*.

EQUIVALENCE THEOREM:  $(AtB) \text{ s } (\{AA\}=\{AB\})$ , (6.5).

ETHICAL: Stipulatively defined as anything that moves the individual towards the *suprarational*.

EVENT: A *dissimilarity* compounded with a *duration*. Also called a *change*.

EVIL: *Hekergy* decrease.

EXCLUSIVELY EXTENSIONAL FUNCTION: An *extensional function* determined by a *contingent function*.

EXCLUSIVELY EXTENSIONAL MEANING: A mathematical symbol, name, or description has exclusively extensional meaning if its meaning is an *extension* and it has no *intensional meaning* and no *exclusively nominal meaning*.

EXCLUSIVELY EXTENSIONAL SCIENCE: Science which is neither *intensional* nor *exclusively nominal*; characterised by the use of statistics and control groups.

EXCLUSIVELY NOMINAL FUNCTION: A *nominal function* whose domain and/or range are null.

EXCLUSIVELY NOMINAL MEANING: A mathematical symbol, name, or description has exclusively nominal meaning if its meaning is a *nominal set* and it has no *extensional meaning*.

EXCLUSIVELY NOMINAL SCIENCE: All false claims to empirical data, laws, or theories.

EXISTENCE: See *mathematical existence* and *real existence*.

EXTENSION: Any plurality, unified by a relation, or not; a *set* which is either a *complete set* or an *incomplete set*. Also called an *extensional set*.

EXTENSIONAL ANY: The use of the word *any* applied to a *contingent set*,  $S$ , meaning a *random* selection of a *member* of  $S$ .

EXTENSIONAL COMPLEMENT: If  $U$  is the universe of discourse and  $S$  is an *intensional set* then the *extensional complement* of  $S$ , symbolised  $S'$ , is  $S' = U - S$ .  $S'$  is such that  $S \cap S' = \emptyset$  and every *member* of  $S$  is not *identical* with each member of  $S'$ , and vice versa. See also *intensional complement*.

**EXTENSIONAL CONNECTIVES:** Those *relations* between *intensional sets* or *extensional sets* which are defined by means of *identity*. See also *intensional connectives*.

**EXTENSIONAL EQUIVALENCE:** *Set identity*, more commonly known as set equality; it is *extensionally valid* if and only if either (i) *set-membership* in *A* is universally membership in *B*, and vice versa: always if  $x \in A$  then  $x \in B$ , and if  $x \in B$  then  $x \in A$ , or (ii) non-membership in *B* is universally non-membership in *A*, and vice versa: always if  $x \notin B$  then  $x \notin A$ , and if  $x \notin A$  then  $x \notin B$ . See also *extensional inference* and *extensional validity*.

**EXTENSIONAL FALSITY:** Non-membership: the statement “*x* is an *A*” is extensionally false if and only if  $x \notin A$ , and it is *extensionally true* if and only if  $x \in A$ .

**EXTENSIONAL FUNCTION:** A set of ordered pairs determined by either an *intensional function* or a *contingent function*, where each pair is composed of an argument of the function and its corresponding value. The set of ordered pairs must be *intensionally complete* if it is determined by an *intensional function*, and it must be *extensionally complete* if it is determined by an *extensional function*.

**EXTENSIONAL INFERENCE:** The inference from the extension *A* to the extension *B* is the relation of *subset*,  $A \subseteq B$ .

**EXTENSIONAL MEANING:** A mathematical symbol, name, or description has extensional meaning if its meaning is an *extension*; and it has *exclusively extensional meaning* if it has no

*intensional meaning* and no *exclusively nominal meaning*: that is, it is an extension that is not *unified* by a relation, a *contingent set*.

**EXTENSIONAL NATURAL NUMBER:** The extensional natural number of an *extension* is the set of all extensions with which it is in one-to-one correspondence. See also *intensional natural number* and *nominal number*.

**EXTENSIONAL NECESSITY:** *Universality*.

**EXTENSIONAL RELATION:** A *subset* of a *Cartesian product*.

**EXTENSIONAL SET:** Any *set* which is either a *complete set* or an *incomplete set*. Also called an *extension*.

**EXTENSIONAL SET THEORY:** Set theory that deals with *extensional sets*: sets which are either *complete* or *incomplete*, without distinction.

**EXTENSIONAL TRUTH:** *Set-membership*: the statement “ $x$  is  $A$ ” is extensionally true if and only if  $x \in A$ , and it is *extensionally false* if and only if  $x \notin A$ .

**EXTENSIONAL VALIDITY:** The extensional inference from  $A$  to  $B$ ,  $A \vdash B$ , is extensionally valid if and only if either (i) *set-membership* in  $A$  is universally membership in  $B$ : always if  $x \in A$  then  $x \in B$ ; or (ii) non-membership in  $B$  is universally non-membership in  $A$ : always if  $x \notin B$  then  $x \notin A$ .

**EXTENSIONALLY COMPLETE:** A *contingent set* is *extensionally complete* in the sense that its extension is in one-to-one correspondence with its *enumeration*.

**EXTENSION OF AN INTENSIONAL SET:** A plurality unified by a relation is an *intensional set*, and the plurality is its *extension*; a set is one, the plurality many.

**EXTREMISM:** The attitude produced by great *degree of dissimilarity* between *oge-lover* and *oge-enemy*.

**EXTRINSIC INTENSION:** An *intension* whose *property set* is an *extrinsic property set*, or a *subset* of an extrinsic property set.

**EXTRINSIC NECESSARY EXISTENCE:** The *extrinsic property of necessary existence*. See also *compositional existence*, *distributive existence*, and *intrinsic necessary existence*.

**EXTRINSIC PROPERTY:** If the existence of a *relation* R is *skew-separable* from the existence of another relation, S, then R is a *lower extrinsic property* of S, and S, with or without some of the other terms of S, is an *upper extrinsic property* of R.

**EXTRINSIC PROPERTY SET:** The extrinsic property set of a relation R is the *intensional set* of every *upper extrinsic property* of R — that is, every extrinsic property of R other than the terms of R.

**EXTRINSIC SET:** An *intensional set* defined by an *extrinsic intension*.

FEELING: The *ego*'s dynamic *attention* to *values*, as opposed to *thinking* which is its dynamic attention to *sensations* and *relations*.

FEELING OF BEING WATCHED: when committing a solitary *guilty* act, the feeling is due to the *oge* watching.

FUNCTION ANY: The inverse of the *function every*; its arguments are *intensional sets* and its values their *intensions*.

FUNCTION EVERY: The *necessity relation* which has *intensional sets* as its values; its arguments are *intensions*.

FUZZY SET: An intensional fuzzy set, of *degree of fuzziness*  $n$ , is a *similarity set* defined by a *degree of similarity*,  $n$ , to a *property set*  $P$ :  $\{A(t^xP)\}$ , for all  $x$  such that  $n \leq x \leq 1$ .

GENERAL PROBLEM OF PERCEPTION: The problem of deciding whether all that we *empirically perceive* around us is *reality* or images of reality.

GENUINE RELATION: A *relation* which is either *real* or *ideal*. Also called an *intensional relation*.

GENIUS: Anyone with exceptional *creativity*.

GOAL OF THE EGO: Any possibility of its own *hekergy* increase of which the *ego* is *conscious*.

GOOD: *Hekergy* increase, or high *hekergy*.

GOSSIP: A process of *oge* communicating with *oge*.

GRAMMAR: Rules that relate *words* and that relate the *meanings* of words.

GREATER THAN: An *intensional natural number*  $m$  is greater than another,  $n$ , symbolised  $m > n$ , if there exist *representative instances*  $M$  and  $N$  whose *term sets* are such that  $NiM$ . The inverse of greater than is *less than*, symbolised by  $<$ .

GUILT: A feeling produced in the *ego* by the *oge*.

HATE: A willingness to lower the *hekergy* of another person, unconditionally. See also *love*.

HEKERGY OF A RELATION: A measure of the *hekergy*,  $H_R$ , of a *relation*  $R$  is the natural logarithm of the *improbability* of  $R$ ,  $\ln(t/e)$ , where  $e$  is the number of *arrangements* of  $R$  in which  $R$  emerges and  $t$  is the total number of possible arrangements.

HOLY GHOST: The *mind hekergy principle*.

HUMAN VALUES: The *ego's consciousness* of *absolute values*. Also called *empirical values*.

HYPNOTISM: Abdication of the control of the body, by the *ego*, in favour of the *oge*.

IDEAL RELATION: A *relation* which exists within a *theoretical mind*.

IDENTICAL: Two or more symbols, words, names, or descriptions which between them have only one reference are identical.

IDENTICAL SETS: Supposedly two *intensional sets*,  $S$  and  $T$ , are identical, symbolised by  $S=T$ , if every *member* of  $S$  is *identical* with a member of  $T$ , and vice versa.

IDENTIFICATION: What occurs when an *oge-person*, or group of them, becomes joined to the *ego*.

IqEqN: *Intensional meaning* is a sufficient condition for *extensional meaning*, which is a sufficient condition for *nominal meaning*, but the converses are only necessary conditions.

IMAGINATION: The *ego's* manipulation of, and *consciousness* of, *concrete ideas*.

IMPLICATION THEOREM:  $(A \circ B) \supset (\{AA\} \supset \{AB\})$ , (6.6).

IMPROBABILITY OF A WHOLE: If a *relation*  $R$  is the *top relation* of a *whole* and emerges with  $e$  possible *arrangements* of its terms, and there are  $t$  possible arrangements of its terms altogether, so that the *probability* of the whole defined by  $R$  is the *ratio*  $e/t$ , then the improbability of the whole is the ratio  $t/e$ .

INCLINATION-DUTY CONFLICT: Conflict between the *ego* and the *public* portion of the *oge*. See also *neurotic conflict*.

INCOMPLETE DISJUNCTION: An *intensional commonality*, or disjunction, whose *extension* is an *incomplete*, or *contingent*, *set*.

INCOMPLETE SET: Any plurality of *members* which is not unified by a *relation*, and so is a many rather than a one; a plurality that has no *intension*; an *extension* which can be *enumerated* but which cannot be specified by a rule. Also called a *contingent set*.

INSANITY: The result of extreme *dominance* of *ego* over *oge* or *oge* over *ego*.

INSTANCE OF A RELATION: *y* is an instance of a *relation* *R* if and only if *y* is a *member* of the *similarity set* determined by *R*:  $y \in \{A(tR)\}$ . See also *representative instance*.

INTELLIGENCE: The capacity to rearrange *abstract ideas* *rationally*.

INTENSIONAL COMPATIBILITY: Two *relations* are intensionally compatible if both of them could be *terms* of one relation. See also *nominal compatibility*.

INTENSIONAL COMPLEMENT: The consequents of a *bipossibility relation*; if  $Av(R|S)$  then *R* and *S* are intensional complements of each other, symbolised with a prime: *S* is *R'* and *R* is *S'*; that is, every *member* of *S* – *A* is not *similar* to each member of *S'* – *A*.

INTENSIONAL CONNECTIVES: Those *relations* between *property sets* which are defined by means of *similarity*. See also *extensional connectives*, which are defined analogically by means of *identity*.

INTENSIONAL FALSITY: If a *structure* is a copy, representation, or reproduction of another, and they are *dissimilar*, then their dissimilarity is called the intensional falsity of the copy, relative to the other, or original. Also called *dissimilarity falsity*.

INTENSIONAL FUNCTION: Any *relation* whose *property set* is a *superintension of necessity*.

INTENSIONAL MATHEMATICAL SYSTEM: An *axiom structure* and its *emergent structure* together are an intensional mathematical system, or *mathematical system* for short.

INTENSIONAL MEANING: A mathematical symbol, name, or description has intensional meaning if its meaning is a *relation*, or is one or more *intrinsic or extrinsic properties of a relation*.

INTENSIONAL NATURAL NUMBER: An *adicity* (with the exception of the *intensional natural number one*). See also *extensional natural number* and *nominal number*.

INTENSIONAL NATURAL NUMBER ONE: The *commonality* of every *intrinsic property set*, the *commonality* of every *relation*.

INTENSIONAL PROPOSITION: A *structure of ideal relations* — of *abstract ideas*

INTENSIONAL RELATION: A *relation* which is either *real* or *ideal*. Also called a *genuine relation*.

INTENSIONAL SCIENCE: Science whose theory is expressed with *intensional mathematics*.

INTENSIONAL SET: Any plurality unified into a totality by a *relation*. Every intensional set possesses an *intension*.

INTENSIONAL SET THEORY: Set theory that deals with *intensional*, or *complete*, sets only.

INTENSIONAL TRUTH: If a structure is a copy, representation, or reproduction of another, and they are *similar*, then their similarity is called the intensional truth of the copy, relative to the other, or original. Also called *similarity truth*.

INTENSIONALLY VALID INFERENCE: The inference from A to B is an intensionally valid inference if and only if  $UA \supset UB$  or  $UA \supset UB$ .

INTENSION OF A SET: That *property set*, *intrinsic* or *extrinsic*, that all and only the *members* of that *set* have.

INTERSECTION OF INTENSIONAL SETS: The intersection of two *intensional sets*, *S* and *T*, symbolised by  $S \cap T$ , if it exists, is such that every *member* of  $S \cap T$  is *identical* both with a member of *S* and with a member of *T*.

INTRANSITIVE: A dyadic *relation*, *R*, is *transitive* if, given  $Rab$  and  $Rbc$ , it is true that  $Rac$ , and it is *intransitive* if it is not *transitive*.

INTRINSIC INTENSION: An intrinsic intension specifies a *property set* that is *intrinsic* to each *member* of the *intensional set* that it defines, and that set is called an *intrinsic set*.

INTRINSIC NECESSARY EXISTENCE: A relational *property* such that if a *relation* R possesses it, R *exists* necessarily.

INTRINSIC PROPERTY: A property of a relation is an intrinsic property of that relation if the relation and the property are inseparable

INTRINSIC SET: A set defined by an *intrinsic intension*.

INTROJECTION: The incorporation of a *proposition* into the *oge*, thereby making it an *oge-belief*, as a result of another person asserting this proposition in the hearing of the *ego*.

INVERSE: The inverse, S, of a dyadic *relation* Rab is Sba.

IRRATIONAL, THE: Anything in a *theoretical mind* that is structured by *L.A.L.*

JEALOUSY: *Envy* of another's receipt of *love*.

KINDS OF RELATIONS: Kinds of *relations* are distinguished by *enumeration* of their *property sets*.

L.A.L.R.U. or L.A.L.: Abbreviations for like-attract-like-and-repel-unlike.

**LARGER RATIOS:** An *intensional ratio*  $a:b$  is larger than another intensional ratio  $c:d$  if and only if  $ad > bc$ , and  $a:b$  is *smaller* than  $c:d$  if and only if  $ad < bc$ .

**LESS THAN:** An *intensional natural number*  $m$  is less than another,  $n$ , symbolised  $m < n$ , if there exist *representative instances*  $M$  and  $N$  whose *term sets* are such that  $M \text{ in } N$ . The inverse of *less than* is *greater than*, symbolised by  $>$ .

**LEVEL OF A RELATION:** The terms of a *relation*  $R$ , which are themselves relations, are said to be one level lower than the level of  $R$ . The lowest level, level-1, consists of *prime relations*, also called *separators*.

**LINEAR SEPARATOR:** A *separator* that has unit length and is symmetric; also called an *atomic length*.

**LOVE:** A willingness to raise the *hekerger* of another person, unconditionally. See also *hate*.

**LOWER EXTRINSIC PROPERTY:** If the existence of a *relation*  $R$  is *skew-separable* from the existence of another relation,  $S$ , then  $R$  is a lower extrinsic property of  $S$ , and  $S$ , with or without some of the other terms of  $S$ , is an *upper extrinsic property* of  $R$ ; in ordinary language,  $R$  is a term of  $S$ .

**MAGIC:** Sometimes an attempt to control the *oge*.

**MALICE:** *Ego* harm to the *oge*.

MANIC-DEPRESSIVE: A person who oscillates between high *ego-dominance* and high *oge-dominance*.

MAPPING: A *theoretical idea* is mapped into another idea when a reproduction of it is produced as that other idea.

MATERIAL, THE: Everything in the *consciousness of the ego* that is external, public, reperceptible, and causally coherent.

MATERIAL WEALTH: The emergent relation between the *hekergy* of a *material object* and an *empirical need* of the possessor of the wealth. See also *wealth of a system*.

MATHEMATICAL ENTITY: Intensionally, a mathematical entity is any *intensional meaning*.

MATHEMATICAL EXISTENCE: If a *mathematical entity* A exists, and a mathematical entity B is self-consistent and compatible with A, then B exists; that is, B exists if B is both intrinsically possible, and extrinsically possible relative to A. Conversely, if B is either not self-consistent, or incompatible with A, B does not exist.

MATHEMATICAL SYSTEM: An axiom structure and its emergent structure together are a mathematical system. Also called an *intensional mathematical system*.

MATURATION: *Hekergy* increase of *agents*.

MEMBER: A member of an *intensional set* is any one element of the unified plurality that is the set; the *relation* between a member and its set is the relation of *set-membership*.

MENTAL HEALTH: Harmony between *ego* and *oge*.

MENTAL, THE: Everything in the *consciousness* of the *ego* that is not *material*.

MID-BODY: The *mid-object* that is an image of the *theoretical* body of the person who is *theoretically perceiving*.

MID-MEMORY: A relatively permanent image of a *mid-world*, or a part of a mid-world, *mapped* from a mid-world.

MID-OBJECT: A level-two *structure* in a *theoretical mind*, a structure of *theoretical sensations*.

MID-SENSATION: A level-two *structure* of *atomic ideas*, brought into the *theoretical mind* by *theoretical perception*. Also called a *theoretical sensation*.

MID-WORLD: A *complete structure* of *mid-objects*.

MIND HEKERGY PRINCIPLE: The principle that a *theoretical mind* changes so as to increase *hekergy*, or, if this is not possible, so as to maintain it, or, if this is not possible, so as to minimise its loss.

MORAL, THE: Any increase in the *hekergy* of society, and hence of the *oge*.

MOTOR-IDEA: A *mid-idea* that may be sent down the efferent nervous system so as to produce a specific movement of muscles in the theoretical body.

MULTIPLICATION: The multiplication of two *intensional natural numbers*,  $m$  and  $n$ , symbolised  $m \times n$  or  $mn$ , is the repeated *addition* of  $n$  instances of  $m$ :  $m \times n = m_1 + m_2 + \dots m_n$ .

NATURAL SET: Every *relation* determines two natural sets; the natural set of its terms and the natural set of its properties. These are its *term set* and its *property set*, respectively.

NECESSARY EXISTENCE See *intrinsic necessary existence* and *extrinsic necessary existence*.

NECESSARY SET: Another name for an *intensional set*, every one of which has a *necessary set-membership* and an *intension*. Also called a *complete set*.

NECESSARY SYSTEM: A *mathematical system* having a *top relation*. The existence of the top relation *extrinsically necessitates the existence* of everything else in the system.

NECESSITATES: If  $A$  is the *antecedent* of a *possibility relation* and  $B$  is its single *consequent*, we say that  $A$  necessitates  $B$ . Also a *distributive property* of a relation  $R$  necessitates the existence of that property in each of its terms, and a *compositional property* of a term of a relation  $S$  necessitates the existence of that property in  $S$ .

NECESSITY: Any *possibility relation* having a *degree of possibility* of one, a singular possibility.

NEED: A state in a *theoretical mind* to which the *mind hekerger* principle applies.

NEGATION THEOREM:  $\{AP'\}=\{AP\}'$  and  $\{AP\}=\{AP'\}'$ , (6.11).

NEUROTIC CONFLICT: Conflict between the *ego* and *private* portions of the *oge*. See also *inclination-duty conflict*.

NOMINAL COMPATIBILITY: Two symbols, words, propositions, or systems are nominally compatible if they do not produce a contradiction when combined. See also *intensional compatibility*.

NOMINAL FALSITY: Incorrect statement of fact, as in error or deceit.

NOMINAL FUNCTION: An *extensional function* whose domain and/or range are *nominal sets*.

NOMINAL MEANING: A symbol, name, or description has *nominal meaning* if its meaning is a *nominal set*; and it has *exclusively nominal meaning* if it has no *extensional meaning*.

NOMINAL NUMBER: Any *intensional* or *extensional natural number* or any number defined nominally out of these natural numbers.

NOMINAL RELATION: A grammatical form of words that indicates a relation. See also *purely nominal relation*.

NOMINAL SET: A *set* which is either an *extensional set* or a *null set*.

NOMINAL SET THEORY: Set theory that deals with *nominal sets*.

NOMINAL TRUTH: Correct statement of fact — intensional or extensional, ideal or real.

NOMINAL VALIDITY: A nominal inference of one statement, Q, from another, P, is nominally valid if and only if the truth function  $PhQ$  is tautologous, or always nominally true.

NOVEL PROPERTY: A *property of a relation R* is a novel property if it is not possessed by any *member* of its *term set*, *R*, or any of the *subordinate terms* of *R*.

NULL SET: A *set* which has no *extension*.

OCCAM'S RAZOR: In a mathematical context Occam's Razor may be stated as: do not multiply the entities in the *axiom structure* beyond the necessity of maximising *emergents*. In a scientific or philosophical context it may be stated as: do not multiply the entities in a theory beyond the necessity of explaining the empirical facts. The ontological argument shows these two statements to be equivalent.

**ORDERED SET** A set in which the members are ordered: first, second, third, etc. An ordered set is symbolised by parentheses, such as (a,b), as opposed to an unordered, or regular, set which is symbolised by braces, such as {a,b}. The difference between ordered and unordered is that  $\{a,b\}=\{b,a\}$  but  $(a,b)\neq(b,a)$ . Ordered sets are needed in the definition of *Cartesian product*.

**OGE:** (Rhymes with bogey.) A second *agent* in the *theoretical mind* which grows out of *mid-memories* of other people; as such it represents, and acts on behalf of, society.

**OGE-COMPULSION:** The forcing, by the *oge*, of an *action* against the *will* of the *ego*.

**OGE COMPULSIVE FAILURE:** An *oge compulsion* for the *ego* to fail, as a result of a parent denying the ego the opportunity to *mature*.

**OGE-GOD:** The *oge*.

**OGE-DOMINANT TYPE:** A person whose *oge* is stronger than their *ego*.

**OGE-ENEMY:** The *oge-person* who *hates* the *ego*.

**OGE INFERIORITY COMPLEX:** A *prejudice*, in the *oge*, that the *ego* is inferior.

**OGE-INHIBITION:** The prevention, by the *oge*, of an *action* willed by the *ego*.

OGE-LOVER: The *oge-person* who loves the *ego*.

OGE-PERSON: A set of *mid-memories* of one person, in the *oge*.

OGE SUPERIORITY COMPLEX: A *prejudice*, in the *oge*, that the *ego* is superior.

ORDINARY THOUGHT: Manipulation of *abstract ideas* with the aid of language. See also *pure thought* and *calculation*.

ORDINATE TERMS: The immediate *terms* of a *relation*, as opposed to its *subordinate terms*.

OWNERSHIP: An emergent *relation* between a *theoretical* person or group of theoretical people, who are the owner, and a theoretical object; characterised by control of the object by the owner, and lack of such control by non-owners.

PAIN: The *ego's consciousness* of *hekergergy* decrease; *pleasure* is its consciousness of *hekergergy* increase.

PANACEA GOD: A *prejudice*.

PATTERN DIAGRAM: The intensional equivalent of a Venn diagram.

PARTS OF A WHOLE: A *whole* is any *unifying relation* *R* which possesses at least one property not possessed by any *member* of its *term set*, *R*, together with *R*, and the *arrangement*, *A*, of *R*. The members of *R* are called the parts of the whole.

PATCH MAPPING: A *mapping* of a minimal area of a *concrete* original, just large enough to be within *consciousness*.

PERSONA: A Jungian *archetype*, the *top ego*.

PHILOSOPHER'S GOD: The *theoretical world*.

PLEASURE: The *ego's consciousness* of *hekergy* increase; *pain* is its consciousness of *hekergy* decrease.

POORER: If one *mathematical system* has less *wealth* than another it is poorer than the other.

POSSIBILITY RELATION: A *relation* which is characterised by having one special term called the *antecedent*; all its other terms are called *consequents*.

PREJUDICE: A *structure* consisting of a *belief* and supporting evidence for that belief; the belief attracts evidence in favour of itself and repels evidence against itself, by *L.A.L.*

PRIME AXIOM STRUCTURE: An *axiom structure* of which the lowest *level* relations are *prime relations*.

PRIME LEVEL SUBORDINATE ADICITY: The lowest level *subordinate adicity* of a relation.

PRIME RELATIONS: *Relations* at the lowest *level*; also called *separators*.

**PRIVATE BY DISSIMILARITY:** The observations of two people are private by dissimilarity if they have a high *degree of dissimilarity*. See also *public*.

**PRIVATE BY PLURALITY:** The observations of two people are private by plurality if they are not *public by identity* — if the observations are two, not one. See also *public*.

**PROBABILITY:** If an *antecedent*  $a$  of a *contingency relation* of *degree*  $c$  has as one of its *consequents* the *term*  $c$ , and each contingency is weighted equally, then the probability of  $c$ , given  $a$ , is  $1/c$ ; and if there are  $n$  of the possibilities that are equivalent for some reason then the probability of this equivalence, given  $a$ , is  $n/c$ .

**PROBABILITY OF A WHOLE:** If a *unifying relation*,  $R$ , of a *whole emerges* with the number  $e$  of *arrangements* of its *terms*, and there are  $t$  possible *arrangements* of its terms altogether, the probability of the whole defined by  $R$  is the ratio  $e/t$ .

**PROCESS:** A sequence of *changes*.

**PROJECTION:** Filtration of *empirical perception* through the *oge*.

**PROPER NAME:** A *word* bonded to a single object.

**PROPERTY OF A RELATION:** The properties of a *relation*  $R$  are the *terms* of *similarity* and *dissimilarity* relations.

PROPERTY SET: When used without qualification it is the *natural set* of the *intrinsic properties* of a *relation*. The *intrinsic property set* of a of a relation R is the *intensional set* of every intrinsic property of R; it is symbolised by the same letter, as a capped small cap: R. The *extrinsic property set* of a of a relation R is the intensional set of every *upper extrinsic property* of R.

PROPOSITION: A *structure* of *abstract* and/or *concrete ideas*.

PSYCHOHELIOS: that part of the *ego* that is wholly *rationally* ordered; the god of truth.

PSYCHOPATH: A person with little or no *oge*. Also called a *sociopath*.

PUBLIC BY IDENTITY: The observations of two people are public by identity if they both observe one and the same thing. See also *private*.

PUBLIC BY SIMILARITY: The observations of two people are public by similarity if they both observe numerically distinct things which have a high *degree of similarity* between them. See also *private*.

PUBLIC RITUAL: A process of obtaining *oge approval* of the object of the ritual.

PURE THOUGHT: Manipulation of *abstract ideas* (*relations* and their *properties*, *intensional meanings*) without associated language. See also *ordinary thought* and *calculation*.

PURELY EXTENSIONAL MEANING: A mathematical symbol, name, or description has purely extensional meaning if it has *extensional meaning*, no *intensional meaning*, and no *exclusively nominal meaning*; that is, its only meaning is a *contingent set*.

PURELY INTENSIONAL MEANING: A mathematical symbol, name, or description has purely intensional meaning if it has *intensional meaning*, no *exclusively extensional meaning* and no *exclusively nominal meaning*.

PURELY NOMINAL RELATION: A *nominal relation* that has no reference.

PURITAN: A person for whom all *pleasure* is *evil*.

RANDOM: Some *degree of contingency*.

RATIO: There is a possibility relation called a *comparison of intensional numbers*, having a *degree of contingency* of three, whose *antecedent* is always an ordered pair of intensional natural numbers, and whose consequents are *larger than*, *equal*, and *smaller than*. The *comparison* of two numbers, *m* and *n*, is then the intensional ratio of those numbers, symbolised *m:n*, or, more conventionally, *m/n*.

RATIONAL, THE: Anything in a *theoretical mind* that is structured with maximum *hekerger*.

RATIONALISATION: The manufacture, by the *ego*, of a false belief that supports *vanity*.

REAL EXISTENCE: All that exists independently of anyone's mind.

REALISM: The belief that the *empirically real* is *theoretically real*; loosely, the belief that empirical objects continue to exist between occasions of being perceived. Also called *common sense realism*.

REALITY: See *empirical reality* and *theoretical reality*.

REAL RELATION: A *relation* which exists independently of any *theoretical mind*.

REFLEXIVE RELATION: A dyadic *relation* R is reflexive if, of any possible *term* a of R, it is true that Raa. R is not monadic in such a case because the expression Raa means that two distinct *instances* of a are the terms of R. If a is one specific instance in Raa, then Raa is monadic and so a *purely nominal relation*.

RECOGNITION: The result of the *comparison* of a present perception with a memory: if the comparison yields *similarity* then recognition is *consciousness* of the similarity, while if the *comparison* yields *dissimilarity*, no recognition occurs.

RELATION: A primitive concept, relations have two characteristics: they relate terms, and they have properties. The properties of a relation are what determines the *kind* of relation that it is, and the terms determine the *instance* of that kind.

REPRESENTATIVE INSTANCE:  $y$  is a representative instance of a *relation*  $R$  if and only if  $y$  is a *member* of the *superintension set* determined by  $R$ :  $y \in \{A(\text{OR})\}$ . See also *instance*.

RICHER: If one *mathematical system* has greater *wealth* than another it is richer than the other.

RUDENESS ABILITY: An index of *degree of ego-dominance*.

SCALE MAPPING: A *mapping* with uniform enlargement or diminution.

SCHIZOPHRENIC: A person with a strong *oge* and little or no *ego*.

SELFISHNESS: The attitude of the *ego* that results from the *mind hekergy principle*.

SELF-SACRIFICE: The sacrifice of the individual, by the *oge*, for the good of society.

SENTENCE: A set of *grammatically related words*.

SEPARATOR: A *prime relation*; a lowest *level relation*.

SET: See *extensional set*, *intensional set*, and *nominal set*.

SET-DEFINING RULE: A rule specifying the *intension* of a *set*: that is, specifying that *property set* possessed by all and only the *members* of the set.

SET DIFFERENCE: The set difference of two *intersecting intensional sets*, *S* and *T*, symbolised  $S-T$ , if it exists, is the set consisting of those *members* of *S* which are not *identical* with any member of *T*.

SET-MEMBERSHIP: The *relation* between a *member* and its set is the relation of *set-membership*.

SET RELATION: A *relation* which has only the one *intrinsic property* of an *adicity*; it is the kind of relation that unifies most pluralities of *members* into *intensional sets*.

SEXUAL NEUROSIS: What occurs when some aspect of sexuality is driven to the *oge-enemy* by parents who regard it as *evil*.

SHAME: A *feeling* produced in the *ego* by the *oge*.

SIMILARITY: A primitive *relation* which, like *dissimilarity*, has *properties* of relations as its *terms*; the *intensional complement* of dissimilarity.

SIMILARITY SET: The *intensional set* consisting of every *relation* having a *property set* similar to a given property set.

SIMILARITY TRUTH: If a *structure* is a copy, representation, or reproduction of another, and they are *similar*, then their similarity is called the similarity truth of the copy, relative to the other, or original. Also called *intensional truth*.

**SIMILAR PROPERTY SETS:** Two *property sets*, or *kinds*, S and T, are similar, symbolised by StT, if every *member* of S is *similar* to a member of T, and vice versa.

**SKEW-SEPARABLE:** The existence of A is skew-separable from the existence of B if A can exist separately from B, but B cannot exist without A.

**SLEEPWALKING:** A state in which the *ego* is in a deep sleep and the conscious *oge* is controlling the body.

**SMALLER RATIOS:** An *intensional ratio*  $a:b$  is smaller than another intensional ratio  $c:d$  if and only if  $ad < bc$ , and  $a:b$  is *larger* than  $c:d$  if and only if  $ad > bc$ .

**SOCIOPATH:** A person with a strong *ego* and little or no *oge*. Also called a *psychopath*.

**STRUCTURE OF A WHOLE:** A *whole* is any *relation* R that has a *novel property*, together with the *term set*,  $R$ , of R and the *arrangement*,  $A$ , of R. The *arrangement*  $A$  is also called the structure of the whole.

**SUBINTENSION OF A PROPERTY SET:** A *property set*, S, is a *subintension* of another property set, T, symbolised SpT, if every *member* of S is *similar* to a member of T, but not vice versa.

**SUBJECTIVE CO-ORDINATE SYSTEM:** It is universally characteristic of *empirical perception* that we each of us perceive from our own viewpoint; the location of this viewpoint is the location of our own empirical body, a location that we describe as

“I, here, now” and which is the origin of a subjective co-ordinate system whose axes are in front of me, behind me, my left, my right, above me, below me, my past and my future.

SUBMERGENCE: The going out of existence of a *relation* and its *properties*.

SUBORDINATE ADICITY: If a relation R has *adicity*  $r$ , then the first *level* subordinate adicity of R is the *sum* of the adicities of each of the  $r$  terms of R: that is, the total *intensional number* of the terms of the terms of R. The second level subordinate adicity of R is the total number of the terms of the terms of the terms of R. The prime level subordinate adicity of R is the lowest level subordinate adicity of R.

SUBORDINATE PARTS: If R is a *top relation* of a *whole* then the *members* of the term set,  $R$ , of R are called the parts of the whole, and the *subordinate terms* of R are called the subordinate parts of the whole.

SUBORDINATE STRUCTURE: The *structure* of a *subordinate part*.

SUBORDINATE TERMS: The terms of the terms of a *relation* R, the terms of their terms, on down to the lowest *level* terms, are together called the subordinate terms of R; as opposed to the *ordinate terms* of R, which are the immediate terms of R.

SUBSET OF AN INTENSIONAL SET: An *intensional set*,  $S$  is a *subset* of another intensional set,  $T$ , symbolised  $SiT$ , if every *member* of  $S$  is *identical* with a member of  $T$ , but not vice versa.

**SUBTRACTION:** The subtraction of an *intensional natural number*  $n$  from another number  $m$ , where  $m > n$ , is the *intensional* binary operation, or *function*, having the ordered pair of them,  $(m, n)$ , as its argument and their *difference* as its value.

**SUM:** If two relations  $M$  and  $N$  have *disjoint term sets*  $M$  and  $N$ , and *intensional natural numbers*  $m$  and  $n$ , and there exists a *relation*  $R$ , of adicity  $r$ , having the term set  $M \cup N$ , then the sum of  $m$  and  $n$ , symbolised  $m+n$ , is the number  $r=m+n$ . If there exists a relation  $S$  of adicity  $s$  whose term set  $S$  consists of  $M$  and a single other relation, then the sum  $m+1$  is the number  $s=m+1$ .

**SUMMATION HEKERGY OF A WHOLE:** The *sum* of the *hekergies* of the *ordinate* and *subordinate terms* of the *top relation* of the *whole*. See also *emergent hekergy of a whole*.

**SUPERINTENSION OF A PROPERTY SET:** A *property set*,  $S$ , is a *superintension* of another property set,  $T$ , symbolised  $S \supset T$ , if every *member* of  $T$  is *similar* to a member of  $S$ , but not vice versa.

**SUPERINTENSION SET OF A PROPERTY SET,  $P$ :** That *intensional set*, every *member* of which is a *superintension* of the *property set*  $P$ :  $\{x: X \supset P\}$ , or  $\{A(\supset P)\}$ .

**SUPERSET OF AN INTENSIONAL SET:** The inverse of *subset*.

**SUPRARATIONAL:** The state of a *theoretical mind* that has maximum possible *emergent hekergy*.

**SYMMETRIC:** A dyadic relation is *asymmetric* if it has a mathematical sense, as a vector has sense, and it is symmetric if it does not have such a sense; a dyadic *relation*  $R_{ab}$  is symmetric if it is *identical* with its *inverse*,  $R_{ba}$ .

**TABOO:** A very strong *oge-inhibition*.

**TEMPORAL SEPARATOR:** A *separator* that has unit duration, and is *asymmetric*. Also called an *atomic duration*.

**TERM OF A RELATION:** A *lower extrinsic property* of a *relation*; with the exception of some empirical relations, a term of a relation is either another relation or a *property of a relation*.

**TERM SET:** The *natural set* of the *terms* of a *relation*.

**THEORETICAL:** The non-*empirical*, anything not known through the senses.

**THEORETICAL CAUSATION:** Relations of *necessity* in the *theoretical world*.

**THEORETICAL CAUSES:** The *antecedents* of *relations of necessity* in the *theoretical world*.

**THEORETICAL EFFECTS:** The *consequents* of *relations of necessity* in the *theoretical world*.

**THEORETICAL MIND:** All the *atomic ideas* in a theoretical brain, plus all innate ideas, plus all data that are brought in by the

theoretical afferent nerves, plus all that emerges *cascadingly* out of these.

**THEORETICAL PERCEPTION:** The scientific explanation of *empirical perception*.

**THEORETICAL SENSATION:** A *level-two* structure of *atomic ideas*, brought into the *theoretical mind* by *theoretical perception*. Also called a *mid-sensation*.

**THEORETICAL REALITY:** Anything that exists independently of being observed.

**THEORETICAL WEALTH:** The emergent *relation* between the *hekergy* of a *theoretical object* and a *theoretical need* of the *ego* of the possessor of the wealth.

**THEORETICAL WORLD:** All that is *theoretically real*.

**THINKING:** The *ego's* dynamic *attention* to *ideas*, as opposed to *feeling*, which is its dynamic attention to *empirical values*. See also *thought*.

**THOUGHT:** The *ego's* manipulation of, and *consciousness* of, *abstract ideas*. See also *imagination*, *calculation*, *ordinary thought*, and *pure thought*.

**TOP EGO:** The portion of the *ego* above the top of the *empirical world*, relative to the *subjective co-ordinate system*; the *persona*.

TOP OGE: The portion of the *oge* above the top of the *empirical world*, relative to the *subjective co-ordinate system*; the *oge-lover*.

TOP RELATION: The relation which unites the *parts of a whole* into a *whole*. The top relation in a *mathematical system* is the one relation the *existence* of which *necessitates* the existence of every one of its *ordinate* and *subordinate terms* and their *arrangements*.

TRANSITIVE: A dyadic *relation*, R, is transitive if, given Rab and Rbc, it is true that Rac. and it is *intransitive* if it is not transitive.

UMBRA: A Jungian *archetype*, the *bottom ego*.

UNION OF INTENSIONAL SETS: The union of two *intensional sets*, S and T, symbolised by  $SgT$ , is such that every *member* of  $SgT$  is *identical* either with a member of S or with a member of T, inclusively.

UPPER EXTRINSIC PROPERTY: If the existence of a *relation* R is *skew-separable* from the existence of another relation, S, then R is a *lower extrinsic property* of S, and S, with or without some of the other terms of S, is an upper extrinsic property of R.

VALUES See *absolute values* and *human values*.

VANITY: A false belief that the *hekergy* of the ego is *greater* than its actual value.

VISIONS: Manifestations in the *ego's empirical world* that originate in the *oge*.

WEALTH, MATERIAL: The emergent relation between the *hekergy* of a *material object* and an *empirical need* of the possessor of the wealth.

WEALTH OF A SYSTEM: If a *mathematical system*  $S$  has a *prime axiom structure*  $A$ , having the number  $a$  of *prime relations*, and an *emergent structure*  $E$  of total *hekergy*  $e$  then the wealth,  $w$ , of  $S$ , is  $w = e/a$ .

WEALTH, THEORETICAL: The emergent relation between the *hekergy* of a *theoretical object* and a *theoretical need* of the possessor of the wealth.

WHOLE: A whole is any unifying relation  $R$  that has a *novel property*, together with the *term set*,  $R$ , of  $R$ , and the *arrangement*,  $A$ , of  $R$ . The *members* of  $R$  are called the *parts of the whole*, and the *arrangement*  $A$  is also called the *structure of the whole*. Any member of  $A$ , of  $R$ , or the sets  $A$  or  $R$  themselves, or  $R$ , is an *element of the whole*, and  $R$  is called the *top relation* of the whole.

WILLING BY THE EGO: The movement of *motor-ideas* to their *action points* by the *ego*.

WORD: A *structure of theoretical and empirical ideas bonded together*, consisting of a *memory* of a sound, the *motor ideas* to produce a *similar* sound, and a *theoretical idea* that is the meaning of the word; it usually also has bonded to it a memory of a written word and the motor ideas to produce a similar written word; and it may have similar *wholes* bonded to it that are

synonyms, and special symbols or foreign words having the same meaning.

# Index

*This index does not refer to entries in the glossary.*

## A

absolute values..... 167  
 abstract  
   idea10, 155, 169  
 action ..... 171  
 action-point..... 171  
 addition..... 79  
 adicity ..... 9  
 affirmation of the antecedent ..... 67  
 agent .....163, 177, 197  
 algorithm ..... 192  
 algorithmic thought..... 174  
 analytic truth, intensional..... 53  
 angle separator..... 87  
 anima ..... 195  
 animus ..... 195  
 Anselm..... 200  
 antecedent of a possibility ..... 12  
 anthropic principle..... 119  
 approval..... 177  
 arbitrariness ..... 41  
 archetypes, Jungian..... 191  
 argument from  
   design . 119  
   externality143  
   illusion 132  
   interpretation 145  
   materiality143  
   misrepresentation 144  
   publicity143  
   qualitative difference 144  
   reperceptibility 143  
 Aristotle..... 132, 199  
 arrangement ..... 62  
 assignment ..... 46  
 association of ideas ..... 155

astrology ..... 191  
 asymmetric ..... 12  
 atomic  
   area .....87  
   change ..62  
   duration 88  
   idea ..... 161  
   length....87  
   vector ....88  
   volume..87  
 attention ..... 167  
   to the public 151  
 attitude of the ego ..... 166  
 Austin ..... 142  
 authoritarian..... 191  
 awareness, ego's..... 165  
 axiom  
   generosity2, 42, 98, 107, 118  
   of addition82  
   structure110, 123

## B

beauty ..... 104  
   of theory124, 158  
 behaviourism ..... 159  
 belief..... 141, 169  
 Berkeley..... 136  
 Big Bang..... 89, 103, 118  
 Big Crunch..... 89  
 bipossibility ..... 13, 14  
 bonded ideas ..... 163  
 bonding ..... 182  
 Boolean algebra ..... 45, 48  
 bottom  
   ego180, 186  
   oge .....181  
 boundary ..... 16, 89

## C

calculation ..... 174  
 cascading emergence 9, 42, 88, 89, 94, 98,  
     104, 105, 110, 207  
 causal necessity ..... 156  
 causation ..... 46, 90  
     Humean 155  
 chance ..... 205  
*change* ..... 16, 62, 89  
 chaos ..... 102  
 classification ..... 170  
 common sense ..... 152  
     realism 130  
 commonality ..... 24  
     of a one-membered set   34  
 comparison  
     a ..... 16  
     of numbers 80  
 compatibility ..... 107  
     intensional 108  
     nominal 108  
 complement  
     extensional 26  
     intensional 14, 26  
 complete  
     disjunction 50  
     set ..... 33  
 completeness  
     extensional 35  
 compositional  
     existence 109  
     property 65  
 compound relation ..... 22, 61  
 compoundable relation ..... 61, 80  
 concept ..... 173  
 concrete ..... 205  
     idea ..... 168  
     meaning 172  
     name ... 173  
     quality 120, 168  
 conflict ..... 178  
     inclination-duty 182  
     neurotic 182  
 conjecture

    intrinsic necessary existence, the  
         ..... 110  
 conjunction  
     theorem 31, 96  
 connectives  
     between sets   22  
     extensional 22, 48  
     intensional 22, 47  
     sentential 47, 48  
     truth-functional 48  
 conscience ..... 178  
 consciousness, ego's ..... 165  
 consequents of a possibility ..... 12  
 consistency ..... 66, 83, 107, 137  
 construction  
     intuitionist 69  
 contingency ..... 13  
 contingent  
     function 46  
     set ..... 33  
     system. 115  
 continuum ..... 86  
 contradiction ..... 42, 158  
 contraposition ..... 71  
 control groups ..... 160  
 correlation ..... 155  
 corresponding  
     elements 67  
     members 93  
 cosmic coincidences ..... 118  
 coupling ..... 25  
 creationism ..... 160  
 criteria of  
     empirical science       124, 125,  
         151  
     good explanation       152  
     theoretical science     124, 126,  
         158  
 criterion of  
     constructability 70

## D

data collection ..... 122  
 death ..... 205

decoupling .....	26
degree of	
contingency	13
dissimilarity	91
ego-dominance	188
falsity....	92
inference	92
membership	91
non-membership	91
oge-dominance	188
possibility	13, 92
<i>similarity</i>	62, 90
similarity of sets	93
similarity of wholes	93
truth .....	91
validity	.92
deified teacher .....	197, 198
delusion .....	190
denial of the consequent .....	67
depression .....	189
Descartes .....	120
design of experiments .....	124
desire and aversion .....	168
difference of two numbers .....	79
dimensional analysis .....	89
disapproval .....	177
discrimination .....	175
disjunction	
complete or incomplete	51
intensional	50
theorem	31, 96
disjunctive	
addition.	71
syllogism	71
disparate.....	24
dissimilarity .....	15, 64, 66
false .....	64
set .....	27
distributive	
existence	66, 109
property	65, 137
divine right of kings.....	195
division of two numbers .....	79

## E

economic forces .....	184
ego .....	165
ego-	
memories	164
ego-	
inhibition	179
ego-	
compulsion	179
ego-	
inferiority complex	185
ego-	
superiority complex	185
ego-	
compulsive failure	186
ego-	
dominant type	187
Einstein .....	89, 159
elation .....	189
electromagnetic ether.....	160
element of a whole.....	64
embarrassment .....	178
emergence.....	9
emergent	
hekergy of a whole	101
level of a property	104
structure	110
empirical	
body.....	205
causation	153
memory	168
object..	165
perception	130
reality	128, 150, 151
science	124, 150
sensation	165
the .....	128
values..	167
world	144, 150, 165
entropy	
of a relation	101
of physics	101
enumeration, an .....	20
envy .....	192
equal	

intensional numbers 78  
 intensional ratios 80  
 equiadic ..... 78  
 equivalence  
     extensional 73  
     intensional 54  
     theorem 31, 95  
 ethical ..... 203  
 Euclid ..... 29  
 event ..... 89  
 excluded middle  
     rule of 69, 70  
 existence  
     compositional 109  
     distributive 66, 109  
     mathematical 107, 114, 118  
     mathematical and real 116  
     necessary 109  
     proofs ... 69  
 explanation, scientific ..... 123  
     intensional 156  
 extension ..... 18, 35  
 extensional  
     analyticity 54  
     *any* ..... 34  
     arithmetic 84  
     complement 26  
     connectives 22, 48  
     equivalence 73  
     falsity .... 72  
     function 46  
     function *any* 34  
     inference 73  
     meaning 38  
     natural number 84  
     necessity 44  
     relation . 11  
     science 159  
     set ..... 35  
     set theory 37  
     truth ..... 72  
 extensionally  
     complete 35  
     valid inference 73, 226  
 extremism ..... 192  
 extrinsic

intension 30  
*necessary existence* 17, 109  
 possibility 107, 108, 124  
 property .. 7  
 property set 19  
 set 30, 61, 206

## F

fallacy of undistributed middle ..... 133  
 falsity  
     dissimilarity 64  
     extensional 72  
     intensional 17, 64  
     nominal . 74  
 Faraday ..... 159  
 feeling of being watched ..... 192  
 feelings and thoughts ..... 168  
 field, physical ..... 89, 106  
 formulation of data ..... 122  
 freedom of the will ..... 206  
 Frege ..... 40  
 function  
     *any*, the . 18  
     contingent 46  
     *every*, the 18  
     extensional 46  
     intensional 45  
     nominal . 47  
 fuzzy  
     set ..... 91  
     set theory and logic 90

## G

Gallop  
     David .... 99  
 generalisation  
     scientific 122  
     superstitious 122  
 generality ..... 41  
 genius ..... 88, 159, 201  
 genuine relation ..... 10  
 geometric points ..... 86  
 goal ..... 168

God .....	119, 120, 197
Gödel's theorems .....	83
good and evil .....	179, 206
goodness .....	104
gossip .....	193
gradient .....	62
grammar .....	173
greater than .....	78
greatest intensional natural number .....	82
guilt .....	178

## H

harmony .....	194
Harvey .....	160
hate .....	180
heaven and hell .....	181
hekerger .....	100, 101
emergent .....	101
summation .....	102
high	
ego .....	180
oge .....	181
Hilbert's program .....	83
holy ghost .....	197
horizon of the moment .....	130, 141
Hubble, Edwin .....	118
human values .....	103, 167
hypnotism .....	193
hypotheses, scientific .....	124

## I

idea, abstract .....	10, 155
ideal	
relation .....	10
idempotence .....	69, 70
identification by the ego .....	181
identity .....	15
error .....	139, 145, 149, 191, 198
rule of ... ..	69
set .....	23
illusions produced by irrationality .....	204
imagination .....	169
implication theorem .....	31, 96

impossibility .....	13
improbability of a whole .....	100
inclination-duty conflict .....	182
incomplete set .....	33
indirect perception .....	141
individuality .....	205
infinite	
divisibility .....	86
enumeration .....	85
extravagance .....	5
intensional numbers .....	82
numbers .....	82, 85
regress .....	86, 110
information .....	102, 131, 184
insanity .....	189
instance of a relation .....	7, 20, 28
integration of theories .....	124
intelligence .....	202
intension .....	18
extrinsic .....	30
intrinsic .....	30
intensional	
analytic truth .....	53
arithmetic .....	78
compatibility .....	108
complement .....	14, 26
conjunction .....	50
connectives .....	22, 47
disjunction .....	50
equivalence .....	54
falsity .....	17, 64
function .....	45, 90
geometry .....	85
mathematics .....	206
meaning .....	38, 169
natural number .....	77
natural number one .....	77
necessity .....	44
negation .....	49
relation .....	10
science .....	159
set .....	18
set theory .....	37
truth .....	17, 64
intensionally	
false .....	64

valid inference	65
interpolation in science	160
interpretation, perceptual	136
intersection	24
intransitive	12
intrinsic	
intension	30
necessary existence	110
need ....	103
possibility	107, 124
property ..	7
property set	19
set .	30, 206
intrinsic necessary existence conjecture	
.....	110
introjection.....	178
intuitionists .....	69, 70
inverse .....	11
inverse square law .....	88, 162
irrational .....	163

## J

jealousy.....	193
Jung .....	191
justice .....	194

## K

kind of relation .....	7, 20
Kronecker .....	84

## L

L.A.L.....	162
law of diminishing returns of wealth ..	111
law, scientific.....	123
laws of thought .....	69
Leibniz.....	118, 120
Leibniz-Russell theory .....	126, 150
less than .....	78
level of a relation .....	80
life	
definition of	103, 125
linear separator .....	86

Locke .....	163
logical necessity .....	156
love .....	180
low	
ego .....	180
oge .....	181

## M

macrostate.....	101
magic .....	193
magnitude of a prime relation .....	85
malice .....	194
manic-depressive .....	190
mapped ideas .....	163
mapping	
boundary	175
patch ...	175
scale....	175
marriage.....	195
material .....	176
implication, paradoxes of	44
wealth .	182
mathematical	
discovery and invention	98, 104
entity... ..	107
existence	107
libraries.	90
metaphysics	118
system.	110
mathematics .....	207
maturation.....	181, 204
meaning	
concrete	172
relational	60
measure.....	85
member .....	18
Mendelieff .....	160
mental .....	176
health ..	194
metaphysics .....	152, 207
methods of science.....	126, 158
microstate .....	101
mid-	
body ....	164

memory 164  
 object.. 164  
 sensation 164  
 world .. 164  
 Mill's Methods ..... 158  
 mind hekergy principle 161, 197, 198, 201  
 minds of mathematicians ..... 90  
 money ..... 184  
 monotheism ..... 197  
 moral..... 178, 203  
 motor-idea ..... 170  
 multiplication of two numbers..... 79  
 myth..... 152

## N

natural number  
     extensional 84  
     intensional 77  
 natural set ..... 18  
 necessary  
     *existence* 17, 109  
     set ..... 33  
     system 116, 239  
 necessity ..... 13, 44, 155  
     causal.. 156  
     intensional 17  
     logical . 156  
 need ..... 162  
 negation  
     intensional 49  
     theorem 31, 97  
 neurotic conflict..... 182  
 Newton ..... 159  
 nominal  
     arithmetic 85  
     compatibility 108  
     connectives 49  
     degree of validity 92  
     falsity .... 74  
     function 47  
     meaning 38  
     necessity 44  
     number . 85  
     relation . 11

science 159  
 set ..... 36  
 set theory 37  
 truth ..... 74  
 nominalists ..... 174  
 nominally valid inference ..... 74  
 non-contradiction, rule of ..... 69  
 novel property ..... 63  
 null set ..... 31, 36  
 number, nominal ..... 85

## O

objectivity ..... 125, 151  
 Occam's Razor .... 5, 24, 39, 114, 119, 206  
     converse of 16  
 oge ..... 163, 177  
 oge-  
     inhibition 179  
 oge-  
     compulsion 179  
 oge-  
     person . 179  
 oge-  
     lover .... 180  
 oge-  
     enemy . 180  
 oge-  
     superiority complex 185  
 oge-  
     inferiority complex 185  
 oge-  
     compulsive failure 186  
 oge-  
     dominant type 187  
 oge-  
     god ..... 197  
 one-to-one correspondence ..... 35, 84  
 ontological argument ... 120, 152, 158, 200  
 order ..... 62  
 orderings ..... 80  
 ordinary thought ..... 174  
 ordinate terms ..... 10

## P

panacea god ..... 197, 199  
 paradox ..... 42, 48  
     Russell's58  
 paradoxes of  
     material implication      44  
 part of ..... 64  
 parts of a whole ..... 63  
 passage of time ..... 206  
 pattern diagram .....42, 55, 153  
 perception  
     empirical130  
     *indirect*141  
     problems of      132  
     substitutes141  
     theoretical131, 144  
 periodic table ..... 160  
 persona ..... 191  
 philosopher's god ..... 197  
 phlogiston ..... 160  
 Planck length and time ..... 88  
 Plato ..... 194  
 pleasure and pain ..... 168  
 Plotinus ..... 199  
 possession  
     of a property by a relation 29  
 possibility ..... 107  
     degree of13  
     extrinsic108  
     relation . 12  
 prediction of  
     novelty123, 126, 153, 156, 160  
     repetition123  
 prejudice ..... 170, 197  
 prime  
     axiom structure 111  
     level .....86  
     relation .85  
 principle of  
     conservation of hekergy   102, 120  
     L.A.L.R.U.162  
     mathematical existence   108  
     mind hekergy   161  
     novel emergence      105, 204

qualitative difference      139, 144,  
     148  
 private by  
     dissimilarity      148  
     plurality148  
 probability .....15, 92, 125, 150  
     of laws and theories      150  
     theory ....15  
 probability of  
     a whole100  
 problem of  
     induction123, 126, 152  
     perception, general      126, 131  
     theoretical knowledge    152  
 problems in  
     philosophy of mathematics      98  
 problems of  
     perception132  
     philosophy of science      125  
 process ..... 90  
 projection ..... 140, 194  
 proper name ..... 172  
 properties of a relation  
     of a relation      8  
 property  
     intrinsic...7  
     lower extrinsic    7  
     novel .....63  
     set .....30  
     upper extrinsic   7  
 property set  
     extrinsic 19  
     intrinsic.19  
 proposition ..... 169  
 prostitute ..... 187  
 psychohelios .....163, 197, 202, 206  
 psychopath ..... 189  
 public  
     ritual ...195  
     space134, 140  
 public by  
     identity147, 198  
     similarity147, 198  
 publicity  
     of belief160  
 pure thought ..... 174

purely nominal relation...5, 11, 13, 36, 40,  
47, 59, 81, 108, 205  
puritan..... 187

## Q

qualitative difference entails quantitative  
difference..... 139

## R

rapist ..... 187  
ratio..... 80  
rational ..... 163  
rationalisation ..... 176  
real  
    existence116  
    relation . 10  
    world .. 116  
realism ..... 130  
reductio ad absurdum..... 69  
reflexive relation..... 12, 70  
relation..... 7  
    *compound*22, 61  
    compoundable 61, 80  
    extensional11  
    genuine . 10  
    ideal ..... 10  
    intensional10  
    monadic..4  
    nominal. 11  
    possibility12  
    purely nominal 11, 13, 36, 37, 40, 47,  
        59, 81, 108, 205  
    real..... 10  
    reflexive12, 70  
    set19, 30, 33, 63, 77  
    top .....63  
    transitive12  
relational meaning .....60, 61, 107  
representative instance ..... 29  
rudeness ability ..... 188  
rules of identity, excluded middle, and  
    non-contradiction ..... 69  
Russell's paradox..... 58

## S

schizophrenic ..... 190  
Schrödinger, Erwin ..... 103, 125  
second law of thermodynamics..... 102  
secondary qualities..... 133, 205  
secondhand ambition ..... 186  
self-  
    membership 59  
    sacrifice196  
selfishness..... 167  
sensation  
    empirical165  
    theoretical164  
sentence ..... 173  
sentential  
    connectives 47  
separator ..... 85  
    angle .....87  
    linear.....86  
    temporal88  
set  
    *contingent*30, 33  
    defining rule 20  
    difference26  
    dissimilarity 27  
    extensional35  
    extrinsic 30  
    identity..23  
    incomplete33  
    intensional18  
    intrinsic.30  
    membership 18  
    natural...18  
    necessary33  
    nominal.36  
    *null*..31, 36  
    one-membered 30  
    property 30  
    relation19, 30, 33, 35, 61, 63, 77, 107  
    similarity20, 30  
    subintension 29  
    superintension 29, 30  
    term .....19  
sexual neurosis..... 186  
shame ..... 178

similarity ..... 15, 66  
     of property sets 23, 64  
     of structures 64  
     of wholes 64  
     set ... 20, 30  
     truth 150, 151  
 simplicity of a  
     theory 124, 158  
 skew-separable ..... 7  
 sleepwalking ..... 196  
 sociopath ..... 189  
 space-time ..... 88, 199  
 speech ..... 172  
 Spinoza ..... 120, 200  
 St. Anselm ..... 120  
 stationary principle ..... 120  
     ultimate 120  
 statistics ..... 160  
 structure of a whole ..... 63  
 subintension ..... 25  
     set ..... 29  
 subjective co-ordinate system ..... 130, 166,  
     177, 180, 205  
 subjectivity ..... 151, 202  
 submergence ..... 9  
 subordinate  
     adicity 80, 82  
     adicity, prime level 81  
     parts ..... 63  
     terms 10, 63, 80  
 subset ..... 25  
 subtraction ..... 79  
 successive approximation to the truth. 124  
 suicide ..... 190  
 sum of two numbers ..... 79  
 summation hekerger of a whole ..... 102  
 superintension ..... 25  
     set ... 29, 30  
 superset ..... 25  
 suprarational ..... 202, 207  
 symmetric ..... 12  
 symmetries  
     within theory 124, 125, 158  
 synthetic truth ..... 53

## T

taboo ..... 179, 198  
 temporal separator ..... 88  
 term  
     of a relation 8  
     set ..... 19  
 theology ..... 152, 207  
 theoretical  
     causation 153  
     mind ... 162  
     perception 131, 144  
     possession or ownership 182  
     prediction of novelty 124, 156,  
         158  
     reality 129, 150  
     science 124, 150, 152, 207  
     sensation 164  
     the ..... 128  
     wealth . 182  
     world 144, 150, 199, 206  
 theory ..... 123  
 thought ..... 169, 174  
     algorithmic 174  
     ordinary 174  
     pure ..... 174  
     rational . 45  
 thoughts and feelings ..... 168  
 top  
     ego ..... 180  
     oge 180, 181  
 trade ..... 102, 183, 184  
 transitive ..... 12  
 truth  
     intensional 17, 64, 103  
     nominal . 74  
     similarity 64, 103  
 truth-functional connectives ..... 48

## U

ultimate  
     scientific theory 118  
 umbra ..... 191  
 underlying causes ..... 123, 152

union.....	25
unit	
magnitude85	
measure 85	
universal characteristic .....	118
universality .....	44, 153

## V

valid inference	
extensionally	73, 226
intensionally	65
nominally74	
values	
absolute167	
human103, 167	
vanity .....	176
velocity of light.....	89
Venn diagrams.....	73, 116
visions, religious.....	196
vitalism .....	160

## W

wealth	
material182	
of a mathematical system 111	
theoretical182	
well-formed formula.....	65
<i>whole</i> .....	22, 63
wholeness of a proposition .....	65
willing of action by the ego .....	171
word.....	172
world	
empirical144	
real.....116	
theoretical144	

## Z

zero .....	81
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*By the same author:*

## Renascent Rationalism

This philosophical work is written for the educated laity rather than the professional philosopher. It covers much of the material of *Relation Philosophy*, but less formally and in greater detail. Beginning with the four kinds of philosophical indubitability — consciousness, indubitable existence, indubitable truth, and indubitable falsity — it deals first with false perception, leading to the Leibniz-Russell theory, and then with false belief, leading to philosophy of science. This in turn leads into metaphysics and relational metaphysics, followed by theory of mind and the use of this theory to separate the trivial from the profound in theology.

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